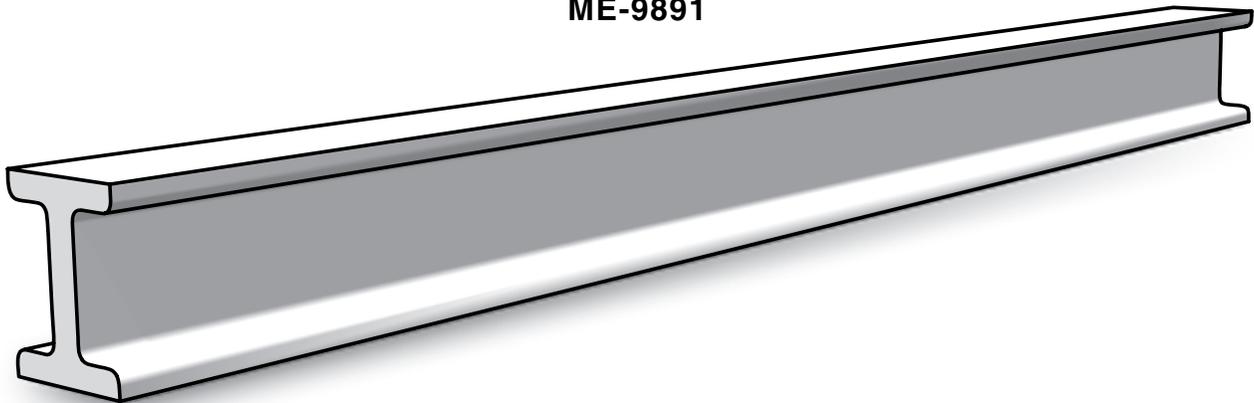


Flexible I-beam

ME-9891



Included Equipment	Part Number
Flexible I-beam	ME-9891
Recommended Equipment¹	
Force Sensor ²	PS-2104
Motion Sensor ²	PS-2103
or	
Rotary Motion Sensor ²	PS-2120
Large Table Clamp	ME-9472
Base and Support Rod	ME-9355
45 cm Rod	ME-8736
Multi Clamp	SE-9442
String (about 1 m)	SE-8050
Small hanging mass (about 50 g)	SE-8759
(2) Pencils	
Adhesive tape (about 20 cm)	

¹Equipment recommended for experiment on page 6.

²Simultaneous use of two PASPORT sensors requires a multi-port PASPORT interface such as Xplorer GLX (PS-2002) or PowerLink (PS-2001); or two single-port interfaces such as USB Link (PS-2100).

Introduction

The model ME-9891 Flexible I-beam illustrates beam stiffness and deflection. Because it flexes more than a metal or wooden beam, students can easily observe that its vertical stiffness is greater than its horizontal stiffness. Use the Flexible I-beam to demonstrate how a beam bends under various load and support configurations. Add sensors to quantitatively explore the concepts of internal moment, second moment of area, and flexural rigidity.

Demonstrations

Ease of Bending

Hold the I-beam in your hands and bend it in the “upright” and “side” directions as illustrated. In which direction is it easier to bend? For the same effort, in which direction does it bend more?

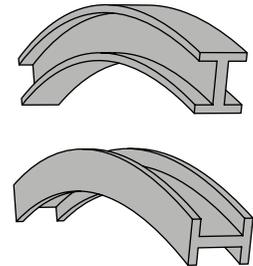


Figure 1: Exaggerated illustration of I-beam bent in upright and side directions

Elastic Curves

Set up the beam (either upright or on its side) in the support-and-load configurations diagramed in Figure 3.

The diagrams show two types of support: roller support, which restricts vertical displacement but not rotation; and fixed support, which restricts both displacement and rotation. Use a pencil for a roller support and a large table clamp (PASCO part ME-9472) for a fixed support (see Figure 2).

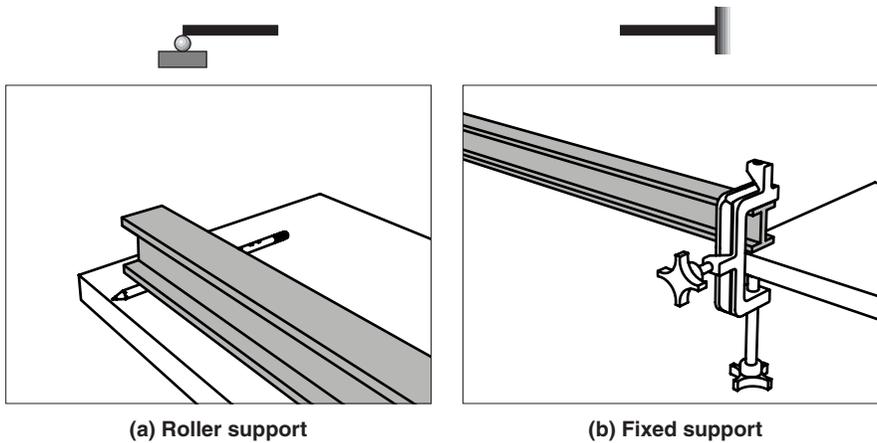


Figure 2

To apply the load, press down on the beam with your hand or place a mass (about 1 kg) on the beam.

For each configuration, make an exaggerated sketch of the elastic curve showing where the curvature of the beam is positive (concave upward), negative (concave downward), and zero (a straight section or an inflection point). Hold a straight edge against the beam to help determine the direction of curvature. Also mark the point (or points) of maximum deflection.

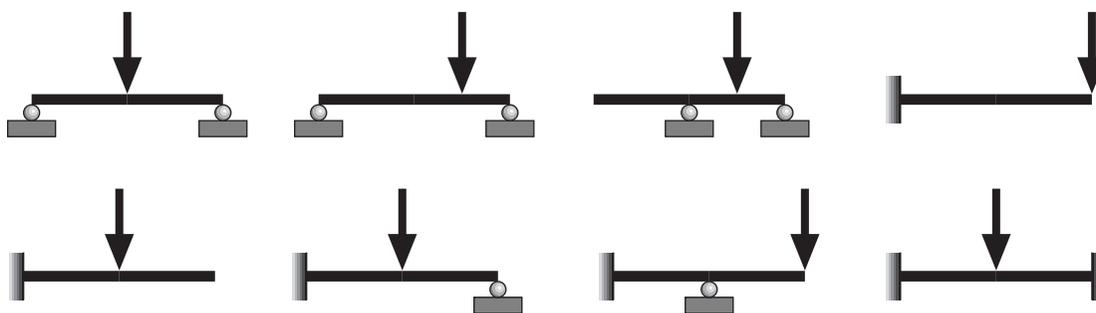


Figure 3: Support-and-load configurations for elastic curve demonstration

Theory

The deflection of a beam depends on how it is supported, the location and magnitude of the load, the beam's elastic modulus (E), and its second moment of area (I).

The vertical deflection of a beam (y) can be found using the formula

(eq. 1)
$$\frac{d^2y}{dx^2} = \frac{M}{EI}$$

where M is the internal bending moment at horizontal position x . The product EI is the beam's flexural rigidity.

Second Moment of Area

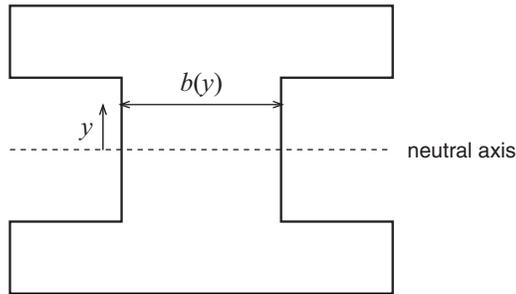


Figure 4

The second moment of area for a beam of constant cross section is

(eq. 2)
$$I = \int y^2 b(y) dy$$

where $b(y)$ is the width of the section at a distance y from the neutral axis (Figure 4).

For the upright I-beam in Figure 5a, the second moment of area is

(eq. 3)
$$I = \frac{WH^3 - wh^3}{12} \quad \text{upright}$$

For the same I-beam turned on its side (Figure 5b), the second moment of area is

(eq. 4)
$$I = \frac{(H-h)W^3 + h(W-w)^3}{12} \quad \text{side}$$

Note that these formulas for I are for calculating *vertical* deflection.

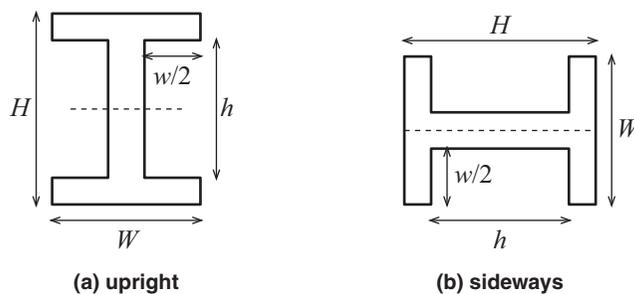


Figure 5

Moment Functions

To predict the deflection of a beam, it is first necessary to express the internal moment M as a function of x along the entire length of the beam. In some cases, the beam must be divided into regions with a different function for each region.

A detailed description of this analysis can be found in most structural analysis text books. Here, we will simply give the solutions for two examples relevant to the experiment below.

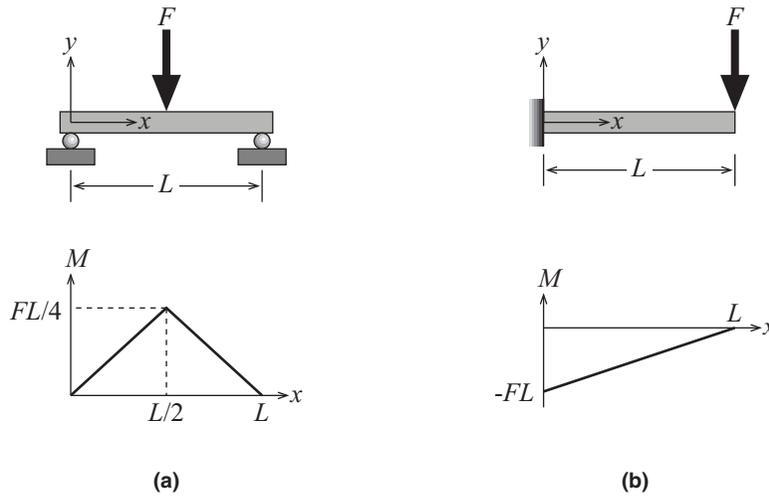


Figure 6

The beam in Figure 6a is supported on two rollers with the load applied in the center. Each support applies an upward force equal to $F/2$. In the *left half* of the beam, the internal moment is $M_1 = Fx/2$ for $0 \leq x \leq L/2$. The internal moment in the *right half* is $M_2 = -F(x - L)/2$.

The cantilevered beam in Figure 6b is supported by a single fixed support. To counter the load, the support applies an upward force equal to F , and a counter-clockwise moment of Fx . The internal moment (for any value of x along the length of the beam) is $M = F(x - L)$.

Deflection Functions

The deflection of a beam under load can be found by combining Equation 1 with the moment function (or functions) and integrating twice to solve for y . Below are two examples of this method used to derive the deflection functions of the beams in Figure 6.

Example 1: Three-point Bending

For the *left half* of the beam in Figure 6a, $M_1 = Fx/2$. Combine that function with Equation 1:

$$\frac{EI}{F} \frac{d^2 y}{dx^2} = \frac{x}{2} \quad 0 \leq x \leq L/2$$

Integrate once:

$$\frac{EI}{F} \frac{dy}{dx} = \frac{x^2}{4} + C_1$$

From symmetry, we know that the slope of the beam at its center is zero: $dy/dx = 0$ at $x = L/2$; therefore $C_1 = -L^2/16$.

Integrate again:

$$\frac{EI}{F} y = \frac{x^3}{12} - \frac{L^2 x}{16} + C_2$$

Because the roller support restricts displacement, $y = 0$ at $x = 0$; therefore $C_2 = 0$, and

$$y = \frac{F}{48EI} (4x^3 - 3L^2 x) \quad 0 \leq x \leq L/2$$

The maximum displacement occurs at $x = L/2$:

$$y_{\max} = \frac{FL^3}{48EI}$$

The right side of the beam can be solved for using Equation 1 and $M_2 = -F(x - L)/2$; however this is not necessary if we are interested only in the maximum displacement at the center of the beam.

Example 2: Cantilevered Beam

To solve the equation for the cantilevered beam in Figure 6b, combine $M = F(x - L)$ with Equation 1:

$$\frac{EI}{F} \frac{d^2 y}{dx^2} = x - L$$

Integrate once:

$$\frac{EI}{F} \frac{dy}{dx} = \frac{x^2}{2} - Lx + C_1$$

Because the fixed support restricts rotation, we know that the slope at the support is zero: $dy/dx = 0$ at $x = 0$; therefore $C_1 = 0$.

Integrate again:

$$\frac{EI}{F} y = \frac{x^3}{6} - \frac{Lx^2}{2} + C_2$$

Because the fixed support restricts displacement, we know that $y = 0$ at $x = 0$; therefore $C_2 = 0$, and

$$y = \frac{F}{6EI} (x^3 - 3Lx^2)$$

The maximum displacement occurs at $x = L$:

$$y_{\max} = \frac{FL^3}{3EI}$$

Experiment: Three-point Bending

Equipment

See Equipment Table on page 1.

Introduction

In this experiment, the I-beam will be supported on two rollers (Figure 7) with a downward load force applied to the center. You will use sensors to measure maximum displacement as a function of applied force. You will repeat the measurement with the beam turned on its side.

The upright beam is more rigid than the sideways beam; therefore, the sideways beam will deflect more.

Before proceeding, predict how much more the sideways beam will deflect (compared to the upright beam) for a given force. Record your prediction and explain how you made it.

Note: The following instructions assume that you are familiar with setting up PASPORT sensors and analyzing data in DataStudio software or on the Xplorer GLX. For more information, see the instructions that came with your sensors and interface, or press F1 to open DataStudio's on-line help.

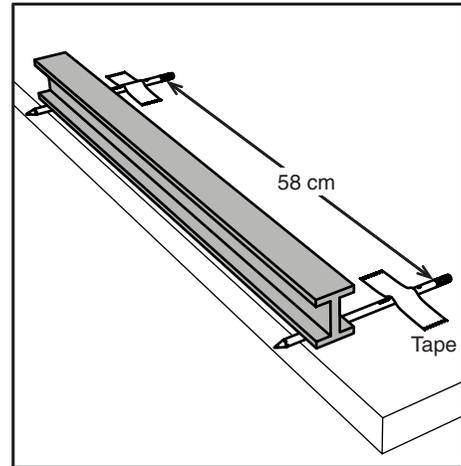


Figure 7: Experiment Set-up

Set-up

1. Place two pencils on your bench top about 58 cm apart (Figure 7).
2. Tape down the pencils to prevent them from moving.
3. Place the I-beam in the upright orientation on the pencils.
4. Set up one of the following sensor options (motion sensor or rotary motion sensor) to measure vertical deflection at the center of the beam.

Motion Sensor option:

- a. Tape a card to the bottom of the I-beam in the center as shown in Figure 8; this card will overhang the edge of the bench and reflect the motion sensor's ultrasonic pulses.
- b. Mount the motion sensor on a rod stand on the floor under the card. Aim the sensor at the bottom of the card. The sensor should be at least 15 cm from the card.
- c. Set the switch on the motion sensor the near-range setting.
- d. Connect the motion sensor to your PASPORT interface.
- e. Record some sample data and ensure that the sensor "sees" the bottom of the card and not the bottom of the lab bench. When you push down on the I-beam, the measured position should change by a few millimeters.

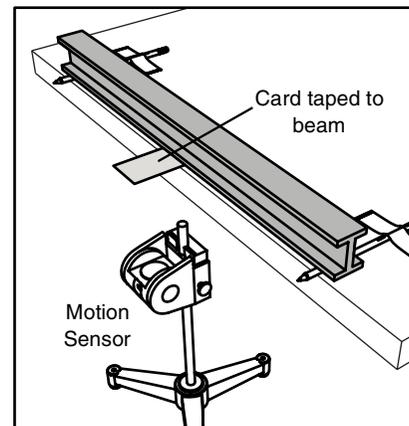


Figure 8: Motion Sensor

Rotary Motion Sensor option:

- a. Install the three-step pulley (included with the sensor) on the rotary motion sensor.
- b. Mount the rotary motion sensor above the I-beam as (shown in Figure 9) using a stand, clamp, and rods.
- c. Tape a piece of string to the center of the I-beam. Run the string over the smallest step of the three-step pulley. Hang a small mass (about 50 g) from the end string.
- d. Connect the rotary motion sensor to your PASPORT interface.
- e. In the **Setup** window of DataStudio (if you are using a computer) or in the **Sensors** screen of the Xplorer GLX (if you are using it in standalone mode), turn on the **Linear Position** measurement and set the **Linear Scale** setting to “Small Pulley (Groove)”.

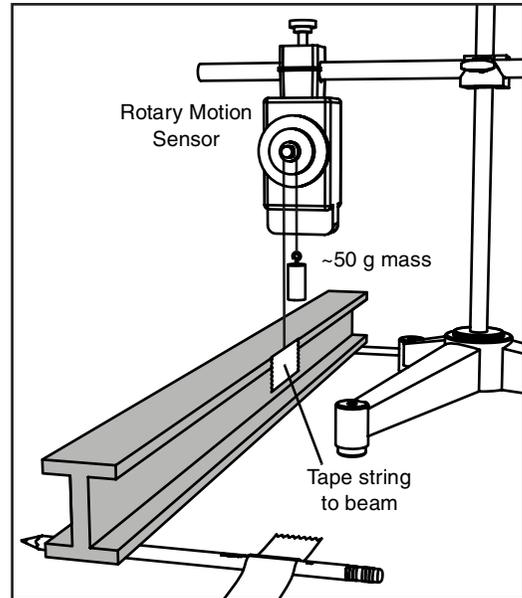


Figure 9: Rotary Motion Sensor

5. Connect a force sensor to your PASPORT interface.

Procedure

1. Start data recording.
2. Hold the force sensor in your hand and push straight down on the center of the I-beam (Figure 10). Gradually increase the applied force until the I-beam has deflected a few millimeters.
3. Stop data recording.

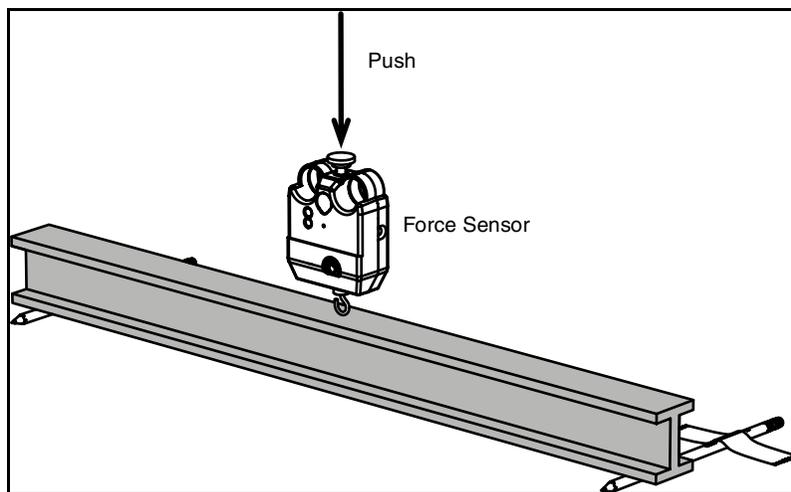


Figure 10: Push down with force sensor

4. Turn the I-beam on its side, adjust the deflection-measuring sensor, and repeat the procedure.

Analysis

1. For the data taken with the beam in the upright orientation, make a graph of y_{\max} (the measured deflection in the center of the beam) versus F (the force applied by the force sensor).
2. Apply a linear fit to the data. The relationship between y_{\max} and F is given by

$$y_{\max} = -F \frac{L^3}{48EI}$$

Therefore, the slope of the best-fit line is $-L^3/(48EI)$.

3. Measure L , the distance between the supports.
4. Use the slope of the best-fit line and L to find EI , the beam's flexural rigidity.
5. Repeat the analysis for the beam in the side orientation.

Questions

1. In which orientation did the beam have greater flexural rigidity? How much greater was it?
2. Was the prediction that you made at the beginning of this experiment accurate?

Further Study

Repeat the experiment with a cantilevered beam (Figure 2b). Does the maximum deflection for a given force change? Does the flexural rigidity change?

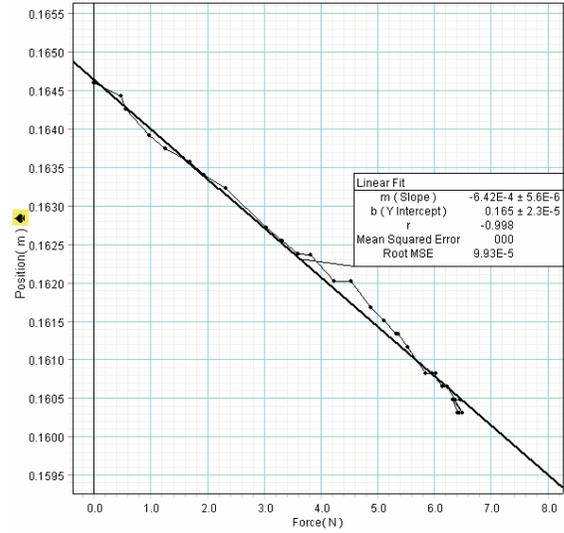
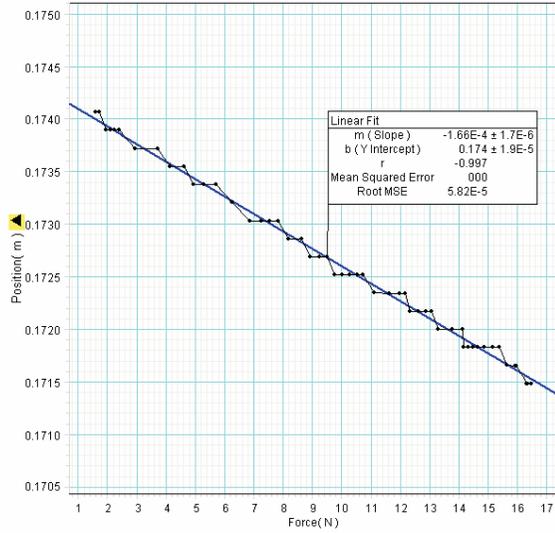
Typical Results

Typical data for the three-point bending experiment are shown below:

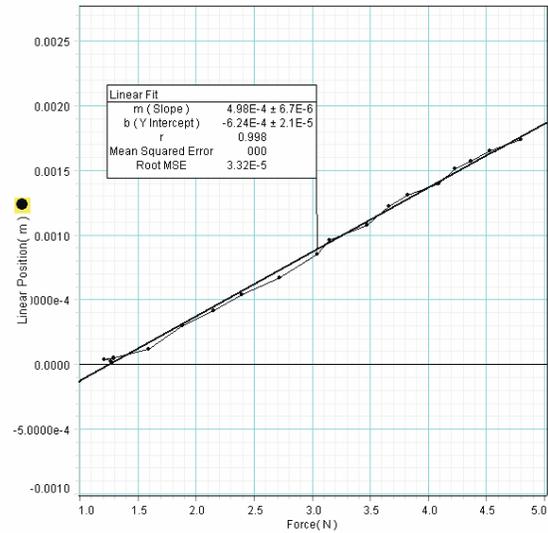
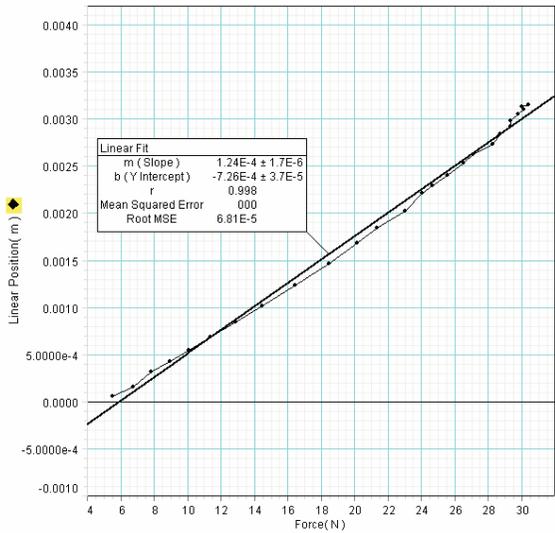
“Upright”

“Side”

Motion Sensor:



Rotary Motion Sensor



Specifications

Length	61 cm (24 inches)
Height	5 cm (2 inches)
Width	4 cm (1.5 inches)

Technical Support

For assistance with any PASCO product, contact PASCO at:

Address: PASCO scientific
10101 Foothills Blvd.
Roseville, CA 95747-7100

Phone: 916-786-3800 (worldwide)
800-772-8700 (U.S.)

Fax: (916) 786-7565

Web: www.pasco.com

Email: support@pasco.com

Limited Warranty

For a description of the product warranty, see the PASCO catalog.

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