Coupled Tanks Control Experiments

33-041S

(For use with MATLAB R2007a or later)

Feedback Instruments Ltd., Park Road, Crowborough, East Sussex, TN6 2QR, UK
Telephone: +44 (0) 1892 653322, Fax: +44 (0) 1892 663719
E-mail: feedback@feedback-instruments.com, Website: www.feedback-instruments.com

Manual: 33-041S Ed01 022009
Feedback Part No. 1160-33041S

THE HEALTH AND SAFETY AT WORK ACT 1974

We are required under the Health and Safety at Work Act 1974, to make available to users of this equipment certain information regarding its safe use.

The equipment, when used in normal or prescribed applications within the parameters set for its mechanical and electrical performance, should not cause any danger or hazard to health or safety if normal engineering practices are observed and they are used in accordance with the instructions supplied.

If, in specific cases, circumstances exist in which a potential hazard may be brought about by careless or improper use, these will be pointed out and the necessary precautions emphasised.

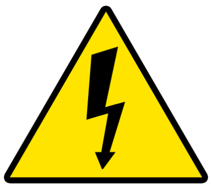
While we provide the fullest possible user information relating to the proper use of this equipment, if there is any doubt whatsoever about any aspect, the user should contact the Product Safety Officer at Feedback Instruments Limited, Crowborough.

This equipment should not be used by inexperienced users unless they are under supervision.

We are required by European Directives to indicate on our equipment panels certain areas and warnings that require attention by the user. These have been indicated in the specified way by yellow labels with black printing, the meaning of any labels that may be fixed to the instrument are shown below:

CAUTION -
RISK OF
DANGER

Refer to accompanying documents



CAUTION -
RISK OF
ELECTRIC SHOCK

CAUTION -
ELECTROSTATIC
SENSITIVE DEVICE

PRODUCT IMPROVEMENTS

We maintain a policy of continuous product improvement by incorporating the latest developments and components into our equipment, even up to the time of dispatch.

All major changes are incorporated into up-dated editions of our manuals and this manual was believed to be correct at the time of printing. However, some product changes which do not affect the instructional capability of the equipment, may not be included until it is necessary to incorporate other significant changes.

COMPONENT REPLACEMENT

Where components are of a ‘Safety Critical’ nature, i.e. all components involved with the supply or carrying of voltages at supply potential or higher, these must be replaced with components of equal international safety approval in order to maintain full equipment safety.

In order to maintain compliance with international directives, all replacement components should be identical to those originally supplied.

Any component may be ordered direct from Feedback or its agents by quoting the following information:

1. Equipment type
2. Component reference
3. Component value
4. Equipment serial number

Components can often be replaced by alternatives available locally, however we cannot therefore guarantee continued performance either to published specification or compliance with international standards.

OPERATING CONDITIONS

WARNING:

This equipment must not be used in conditions of condensing humidity.

This equipment is designed to operate under the following conditions:

Operating Temperature	10°C to 40°C (50°F to 104°F)
Humidity	10% to 90% (non-condensing)

CE DECLARATION CONCERNING ELECTROMAGNETIC COMPATIBILITY

Should this equipment be used outside the classroom, laboratory study area or similar such place for which it is designed and sold then Feedback Instruments Ltd hereby states that conformity with the protection requirements of the European Community Electromagnetic Compatibility Directive (89/336/EEC) may be invalidated and could lead to prosecution.

This equipment, when operated in accordance with the supplied documentation, does not cause electromagnetic disturbance outside its immediate electromagnetic environment.

COPYRIGHT NOTICE

© Feedback Instruments Limited

All rights reserved. No part of this publication may be reproduced, stored in a retrieval system, or transmitted, in any form or by any means, electronic, mechanical, photocopying, recording or otherwise, without the prior permission of Feedback Instruments Limited.

ACKNOWLEDGEMENTS

Feedback Instruments Ltd acknowledge all trademarks.

MATLAB is a registered trademark of Mathworks Inc.

IBM, IBM - PC are registered trademarks of International Business Machines.

MICROSOFT, WINDOWS XP, WINDOWS 2000, WINDOWS ME, WINDOWS NT, WINDOWS 98, WINDOWS 95 and Internet Explorer are registered trademarks of Microsoft Corporation.

Table of Contents

Manual overview

The following manual refers to the Feedback Instruments Coupled Tanks Control application. It serves as a guide for the control tasks and provides useful information about the physical behaviour of the system. A nonlinear model is proposed and identification algorithms are introduced. The models obtained are compared with the phenomenological model and the Coupled Tanks setup. Control algorithms are developed, tested on the models and then implemented in a real-time application.

Throughout the manual various exercises are proposed to bring the user closer to the Coupled Tanks control problem. Depending on the knowledge level of the user some of the sections and exercises can be skipped. The more advanced users can try to model, identify and control the Coupled Tanks on their own from the beginning.

The relative difficulty of each exercise is indicated using the following icons:

-  entry level,
-  medium level,
-  expert level.

If any of the identification or controller design exercises appear to be too difficult or the results are not satisfactory you may go straight to the examples that are supplied and test them by changing the parameters of the controllers.

Introduction

The Coupled Tanks setup is a model of a chemical plant fragment. Very often tanks are coupled through pipes and the reactant level and flow has to be controlled. The Coupled Tanks experiment offers a possibility of system configuration. The couplings between the tanks can be modified to change the dynamics of the system imposing the use of different controllers.

The Coupled Tanks unit allows for the design of different controllers and tests in real -time using Matlab and Simulink environment.

Coupled Tanks set description

The description of the Coupled Tanks setup in this section refers to the mechanical-electrical part and the control aspect. For details on how the signals are measured and transferred to the PC, refer to the '*Installation & Commissioning*' manual.

As shown in Figure 1, the Coupled Tanks unit consists of 4 tanks placed on a rig. Fifth reservoir tank is placed at the bottom. In the reservoir two submersible pumps are placed, which pump the water on command to the tanks. The water flows freely to the bottom tanks through the configurable orifice. The way the water flows through the setup can be configured in many ways with manual valves labelled (MVA, MVB … MVG, MV1, MV2…MV4). Configuration with valves allows for dynamics couplings introduction and step disturbances generation giving vast possibilities of control.

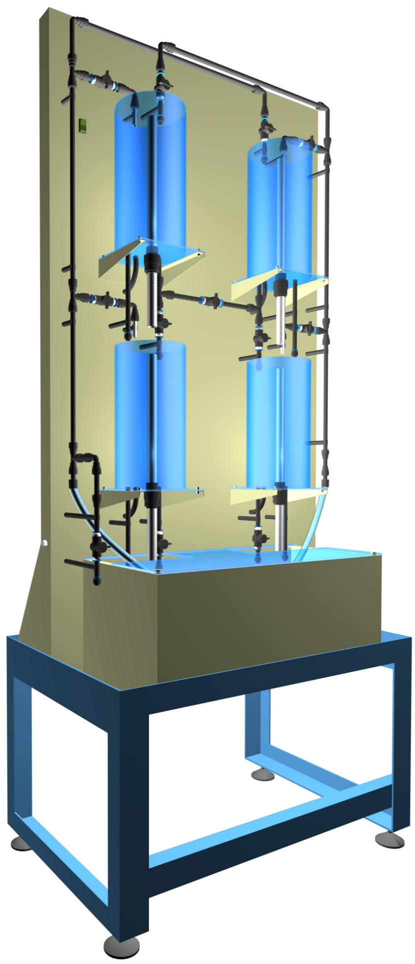


Figure 1 Coupled Tanks mechanical unit

Apart from the mechanical parts the Coupled Tanks system is equipped with Power Supply Unit and Power Amplifier (PSUPA) and the Cable Connector Box (Figure 2). The PSUPA unit amplifies the water pressure-level signals and passes them as analogue signals to the PCI1711 card. The pumps control signal can be sent from the PC through the PCI1711 card and PSUPA unit.

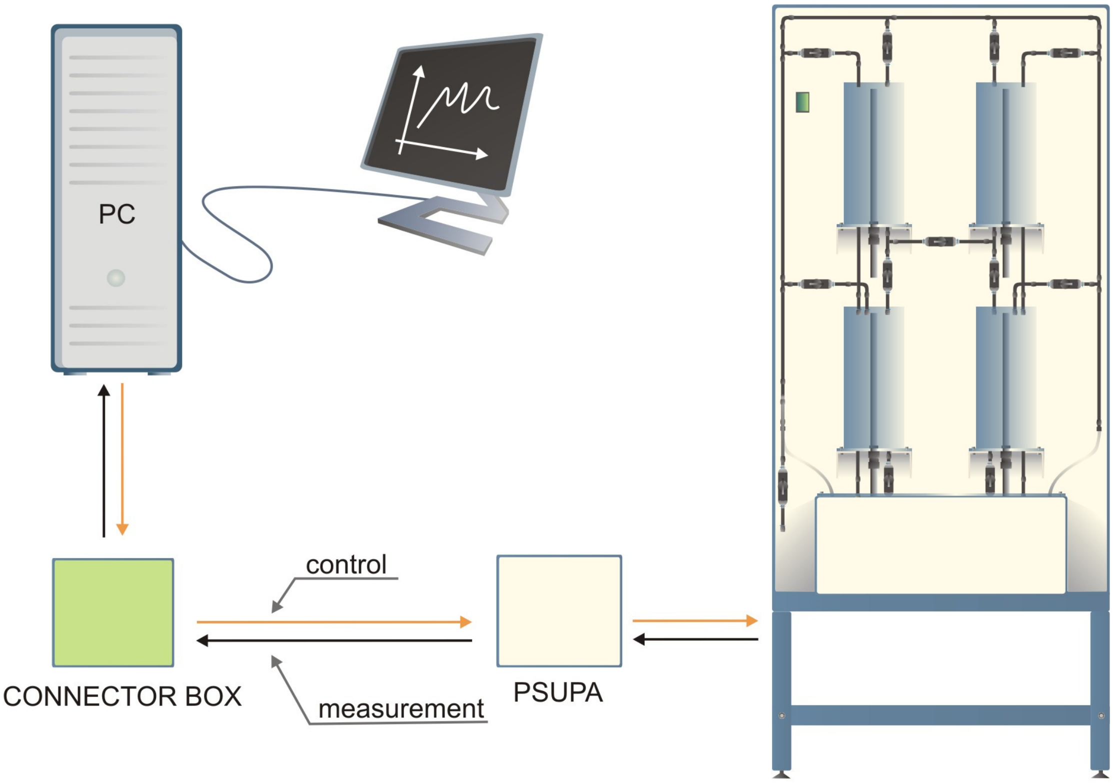


Figure 2 Coupled Tanks control system

In order to design any control algorithms one must first understand the physical background behind the process and carry out identification experiments. The next section explains the modelling process of the Coupled Tank setup.

Two coupled tanks model

Every control project starts with plant modelling, so as much information as possible is given about the process itself. Firstly a single tank and two-tank setup for the modelling task is considered. The time constant of the electrical circuit is significantly smaller from the time constant of the tanks, thus the electrical circuits driving the pump can be treated as an amplification gain in the model. Figure 3 presents the two tank system with its description.

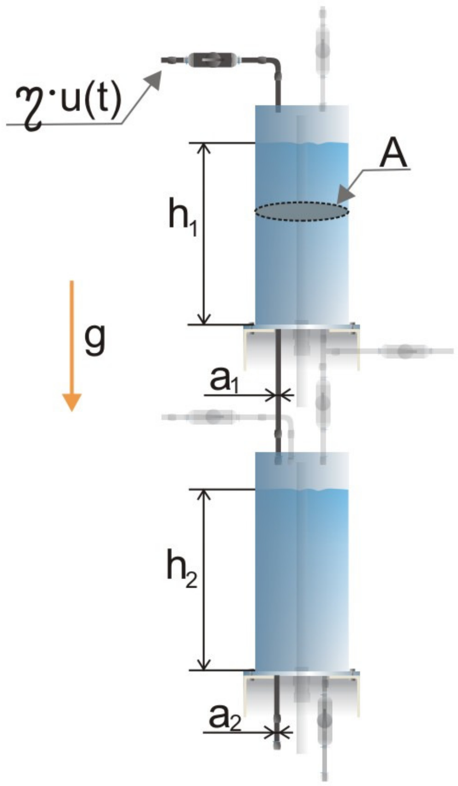


Figure 3 Coupled Tanks phenomenological model

Usually, phenomenological models are nonlinear, that means at least one of the states (*i* – pump driving current, *h*– water level) is an argument of a nonlinear function. In order to present such a model as a transfer function (a form of linear plant dynamics representation used in control engineering), it has to be linearised.

According to the electrical-mechanical diagram presented in Figure 3 the nonlinear model equations can be derived.

The simplest nonlinear model of the coupled tanks system relating the water level h_1 and h_2 with the voltage u applied to the pump is the following:

$$\frac{dh_1(t)}{dt} = -\frac{a_1}{A}\sqrt{2gh_1(t)} + \eta \cdot u(t), \quad (1)$$

$$\frac{dh_2(t)}{dt} = \frac{a_1}{A}\sqrt{2gh_1(t)} - \frac{a_2}{A}\sqrt{2gh_2(t)}, \quad (2)$$

where

h_1 – water level in tank 1, h_2 – water level in tank 2, a_1 – tank 1 outlet area, a_2 – tank 2 outlet area, A – cross-sectional area of the tanks, g – gravitational constant, η – constant relating the control voltage with the water flow from the pump.

Equations (1) and (2) constitute a nonlinear model, which has been assembled in Simulink.

The bound for the control signal is set to [0 .. +5V].

When controlling the water level in the first tank or in the second tank the plant is a SISO plant – single input single output (Figure 4). Water level is the model output and pump control voltage is the control signal (input).

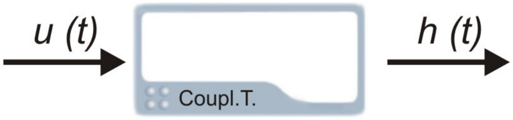


Figure 4 Coupled tanks model

Exercise 1 – Nonlinear model testing

Introduction

For the initial exercise the user has been provided with the nonlinear Coupled Tanks model described by equations (1) and (2). The model can be opened in Simulink - *CT_model.mdl*. It is possible that the assembled phenomenological model responds differently from the Coupled Tanks plant. This might be caused by differences in the model structure itself as well as the value of the chosen parameters. However these differences can easily be detected during the course of identification. One can also modify the phenomenological model by adding gain or by certain parameter tuning.

Load Matlab and the menu system using the Windows Start menu (Figure 5).

Figure 5 Start menu

Matlab will run and the Simulink model menu will open (Figure 6). This contains links to the simulation-only models (Coupled Tanks Simulation Models), the real-time models (Coupled Tanks Real-Time Models) and the manual documents.

Figure 6 Simulink model menu

Double click on the 'Coupled Tanks Simulation Models' block in the main menu. A sub-menu containing all of the simulation-only models will open (Figure 7).

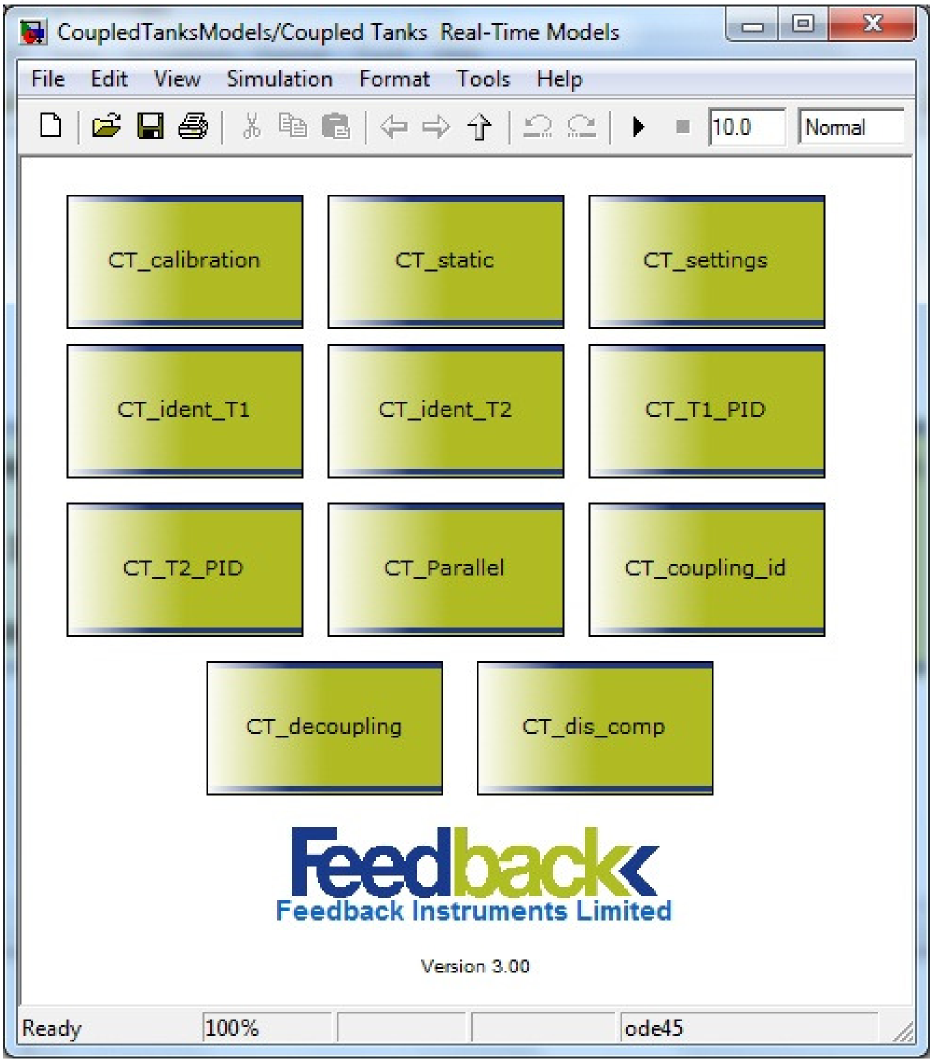


Figure 7 Simulation models menu

Double click the required model *CT_model.mdl*. The Simulink model window will open.

Task

To begin with the user is advised to check the responses of the Coupled Tanks model (*CT_model.mdl)*. Without any control applied the water level will not rise in any of the tanks. Run the test with different values of the control signal by changing the step value. Observe the reaction of the system in both of the tanks under different control signal values.

Example results and comments

Figure 8 presents the response of the Coupled Tanks model when 2.7 V control voltage is applied, which is chosen to be the working point control signal value.

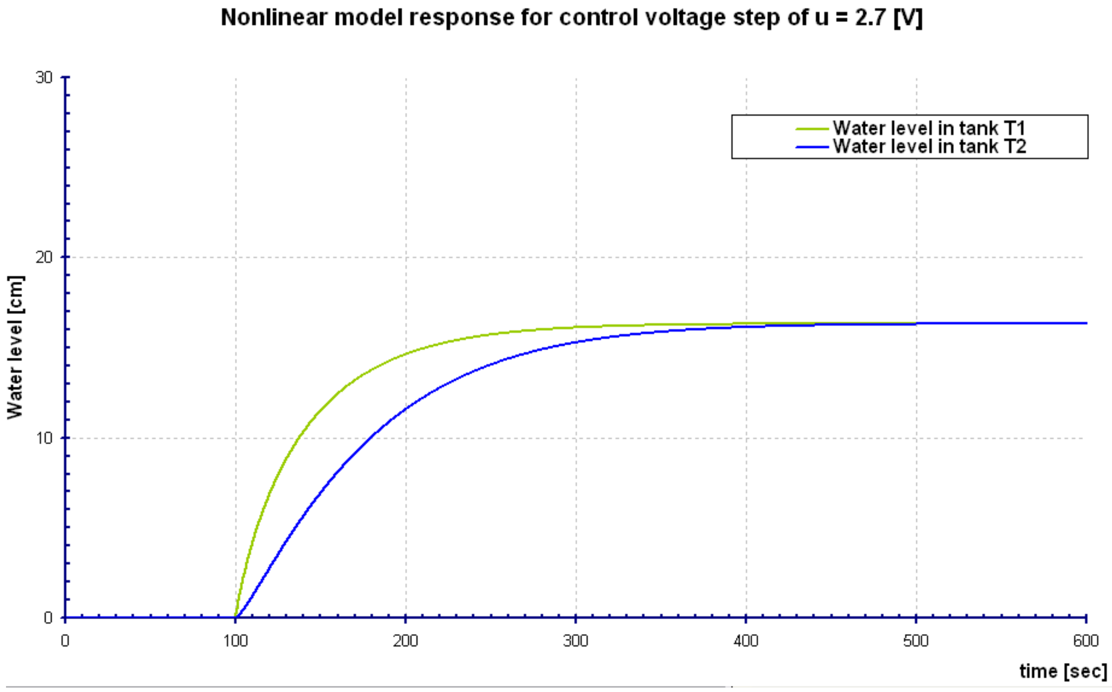


Figure 8 Coupled Tanks first and second tank water level model response

Model linearisation

To carry out analysis of the model dynamics for open loop[1] systems using techniques such as Bode plots, poles and zeros maps, Nyquist plots, root locus (for closed loop[2] systems only), the model has to be linearised. The linearisation is done in the equilibrium point of $h_{1,2} \approx 16\ [cm]$:

Working points calculation:

$$0 = -\frac{a_1}{A}\sqrt{2gh_1(t)} + \eta \cdot u(t), \tag{3}$$

$$0 = \frac{a_1}{A}\sqrt{2gh_1(t)} - \frac{a_2}{A}\sqrt{2gh_2(t)}, \tag{4}$$

$$\begin{aligned} \frac{a_1}{A}\sqrt{2gh_{10}} &= \eta \cdot u_0, \\ a_1\sqrt{2gh_{10}} &= a_2\sqrt{2gh_{20}}, \end{aligned} \quad \Rightarrow \quad \begin{aligned} h_{10} &= \frac{1}{2g}\cdot\left(\frac{\eta u_0 A}{a_1}\right)^2, && (5) \\ h_{20} &= \left(\frac{a_1}{a_2}\right)^2 h_{10}. && (6) \end{aligned}$$

Linearisation:

$$\frac{d\Delta h_1(t)}{dt} = \frac{d}{dh_1}\left(-\frac{a_1}{A}\sqrt{2gh_1(t)}\right) + \eta \cdot \Delta u(t), \tag{7}$$

$$\frac{d\Delta h_2(t)}{dt} = \frac{d}{dh_1}\left(\frac{a_1}{A}\sqrt{2gh_1(t)}\right)\Bigg|_{h10} \cdot \Delta h_1(t) - \frac{d}{dh_2}\left(\frac{a_2}{A}\sqrt{2gh_2(t)}\right)\Bigg|_{h20} \cdot \Delta h_2(t), \tag{8}$$

$$\Delta \dot{h}_1(t) = -\left(\frac{a_1}{A}\right)^2 \frac{g}{\eta u_0} \cdot \Delta h_1(t) + \eta \cdot \Delta u(t), \tag{9}$$

$$\Delta \dot{h}_2(t) = \left(\frac{a_1}{A}\right)^2 \frac{g}{\eta u_0} \cdot \Delta h_1(t) - \left(\frac{a_2}{A}\right)^2 \frac{g}{\eta u_0} \cdot \Delta h_2(t). \tag{10}$$

---

[1] Open loop system – the plant without a controller

[2] Closed loop system – the plant and controller with a negative feedback loop, see “Control” section for more information.

La'place transformation:

$$s\Delta H_1(s) = -\left(\frac{a_1}{A}\right)^2 \frac{g}{\eta u_0} \cdot \Delta H_1(s) + \eta \cdot \Delta U(s), \tag{11}$$

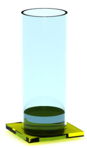
$$s\Delta H_2(s) = \left(\frac{a_1}{A}\right)^2 \frac{g}{\eta u_0} \cdot \Delta H_1(s) - \left(\frac{a_2}{A}\right)^2 \frac{g}{\eta u_0} \cdot \Delta H_2(s). \tag{12}$$

Consequently the respective transfer functions are as follows:

$$\frac{\Delta H_1(s)}{\Delta U(s)} = \frac{\eta}{s + \left(\frac{a_1}{A}\right)^2 \frac{g}{\eta u_0}}, \tag{13}$$

$$\frac{\Delta H_2(s)}{\Delta H_1(s)} = \frac{\left(\frac{a_1}{A}\right)^2 \frac{g}{\eta u_0}}{s + \left(\frac{a_2}{A}\right)^2 \frac{g}{\eta u_0}}. \tag{14}$$

Exercise 2 – Linear model



Introduction

According to the equation (9) a linear model has been assembled in Simulink. You can run similar experiments on it as for the nonlinear model in Exercise 1.

Task

Run the linear (*CT_model_lin.mdl*) and compare the responses. Also, with the use of Matlab, the Bode diagrams, zeros and poles maps can be drawn to carry out initial dynamic response analysis of the Coupled Tanks system.

Example results and comments

Figure 9 presents the response of the Coupled Tanks model when 0.5 V control voltage step is applied.

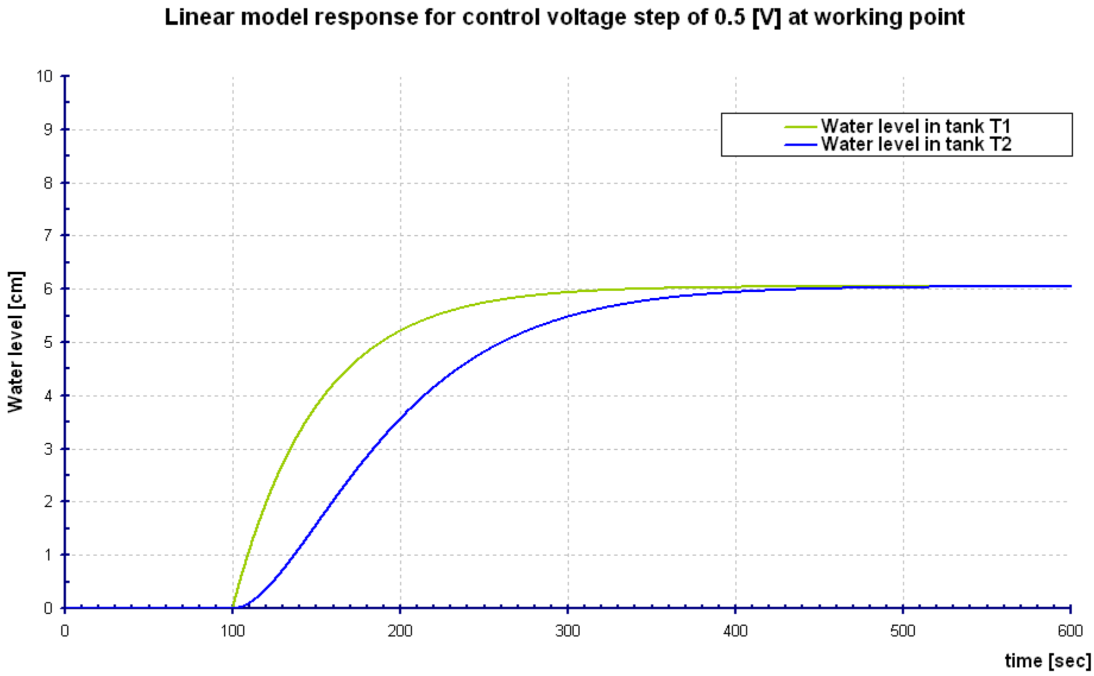


Figure 9 Linear model response

Coupled Tanks model identification

In the previous section a phenomenological model was derived and then linearised. A model can be identified through an identification experiment, upon which controllers will be designed, which is described in the "*Coupled Tanks Control*" section. This section however explains how the discrete, linear Coupled Tanks model identification is carried out.

There are a few important things that the control system designer has to keep in mind when carrying out an identification experiment:

- *Stability problem* – if the plant that is identified is unstable, the identification has to be carried out with a working controller, which introduces additional problems that will be discussed further on. If the plant is stable and does not have to work with a controller the identification is much simpler.
- *Structure choice* – a very important aspect of the identification. For linear models it comes down to choice of the numerator and denominator order of the transfer function. It applies both for continuous and discrete systems.
  As far as the discrete models are concerned the structures are also divided in terms of the error term description: ARX, ARMAX, OE, BJ[3].
- *Sampling time* – the sampling time choice is important both for identification and control. It cannot be too short nor can it be too long. Too short sampling time might influence the identification quality because of the quantisation effect introduced by the AD. Furthermore the shorter the sampling time the faster the software and hardware has to be and more memory is needed. However short sampling time will allow for elimination of aliasing effects and thus anti aliasing filters[4] will not have to be introduced. Long sampling times will not allow for including all of the dynamics.
- *Excitation signal* – for the linear models the excitation choice is simple. Very often designers use white noise however in industrial applications it is often disallowed. It is attractive however because of the fact that it holds very broad frequency content thus the whole dynamics of the plant can be identified. If the dynamics are not too complex several

[3] More information about these structures can be obtained during the System Identification courses.

[4] These are the basics of the Digital Signal Processing course. For more insight the user is asked to study more on signal processing and digital control.

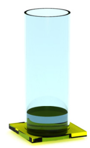
sinusoids with different frequencies can be summed to produce a satisfactory excitation signal.

- *Identification method* – usually two methods are used, the Least Square (LS) method and the Instrumental Variable method. The LS method is the most popular and is implemented in Matlab. This method minimizes the error between the model and the plant output. The optimal model parameters, for which the square of the error is minimal is the result of the identification.

Static characteristic identification

Before any experiments are run it is necessary to define the equilibrium for the Coupled Tanks system. For that purpose a static characteristic of the system has to be identified. The equilibrium can be determined from that static characteristic. Use the *CT_Static.m* to identify the static characteristic. You can use the same m-file to change the working point according to the previously identified static characteristic. Every algorithm will use the working point information for start-up and the control will be performed around the working point water level height and the corresponding control voltage. The simulation is performed automatically. The water height and the control voltage working point values are saved and each of the models will read the working point information.

Exercise 3 – Working point identification



Introduction

Make sure the valves are set-up correctly for this exercise. Open valves MVB and MVE. **The rest of the valves should be kept closed**.

Make sure the unit sensors have been calibrated according to the instructions given in the *Installation & Commissioning* manual.

Task

The experiment lasts **2000 seconds**. Make sure the pumps are properly submersed in the water. Compile and run the model. The working point water levels and control voltages are recorded for both columns. The static characteristic will be recorded. The default working point in terms of water height will be set to 20 cm in the top tank. The voltage will be adjusted according to that level depending on the identified static characteristic. You can change the working point afterwards by running the same CT_static.m file, but omitting the static characteristic identification procedure this time.

Example results and comments

The default working point is saved automatically. The static characteristic is saved and displayed for all 4 tanks in groups of 2. You can look the settings up by executing the CT_Settings.m, by typing CT_Settings in the Matlab command window. The calibration, static characteristic and working point data is displayed.

The static characteristic and working points you obtain might be different from the ones presented in the manuals. This will be due to different water quality and specific setup configuration. Make sure that the chosen working points are not to close to the boundary water level values.

Model identification

The following exercise includes all of the above facts and provides an identification experiment, which results in a discrete and continuous model of the Coupled Tanks unit. The model to be identified describes the relation between the control voltage *u* and the water levels in tank 1 and tank 2 respectively. Two transfer functions are obtained, one relating the pump control voltage with the first tank water level and the second one relating the lower tank water level with the control voltage of the same pump.

Exercise 4 – Model identification

Introduction

Open valve MVB (**leave MVE open – for safety**). The rest of the valves should be kept closed.
Make sure the unit sensors have been calibrated according to the instructions given in the *Installation & Commissioning* manual.

The control real time simulations are carried out with a sampling time of $Ts = 0.1$ [s]. The model identification is carried out with the same sampling time. For the identification the Matlab System Identification Toolbox is used.
The identification experiment is carried out using the model called *CT_ident_T1.mdl* and *CT_ident_T2.mdl*. This models use an excitation signal to vary the value of the control signal, which results in the changes of the water level in the tanks. The experiments last 300 and 1000 seconds and three signals are collected in the form of vectors and are available in the Workspace.

Task

Run the model in Simulink. Make sure the pumps are properly submersed in the water. The water levels and the control signal are recorded.

For instructions on identification experiments refer to the '*Matlab Guide*' manual.

Identify the 2 models using the methods described in the mentioned manual. The suggested orders for the transfer functions are:

- oe111 - suggestion for the first tank model,
- oe221 - suggestion for the second tank model.

Remember to subtract the working point water level and control signal values from the obtained data series.
The obtained discrete model can be transformed into continuous equivalent according to the given instructions. The obtained models will be used in the PID controller design section.

Example results and comments

The obtained transfer functions constitute a model and their dynamics can be compared against the behaviour of the plant. For that purpose step response around the equilibrium point of h = 16 [cm] have been used. The results of the comparison are presented in the following figures.

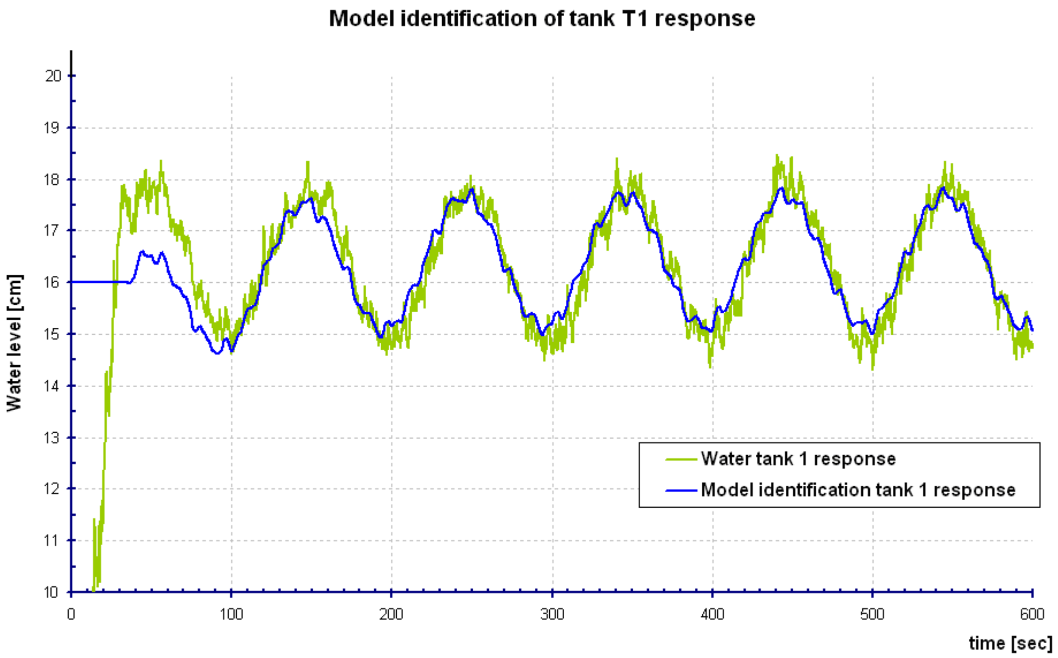


Figure 10 Tank 1 model and plant response

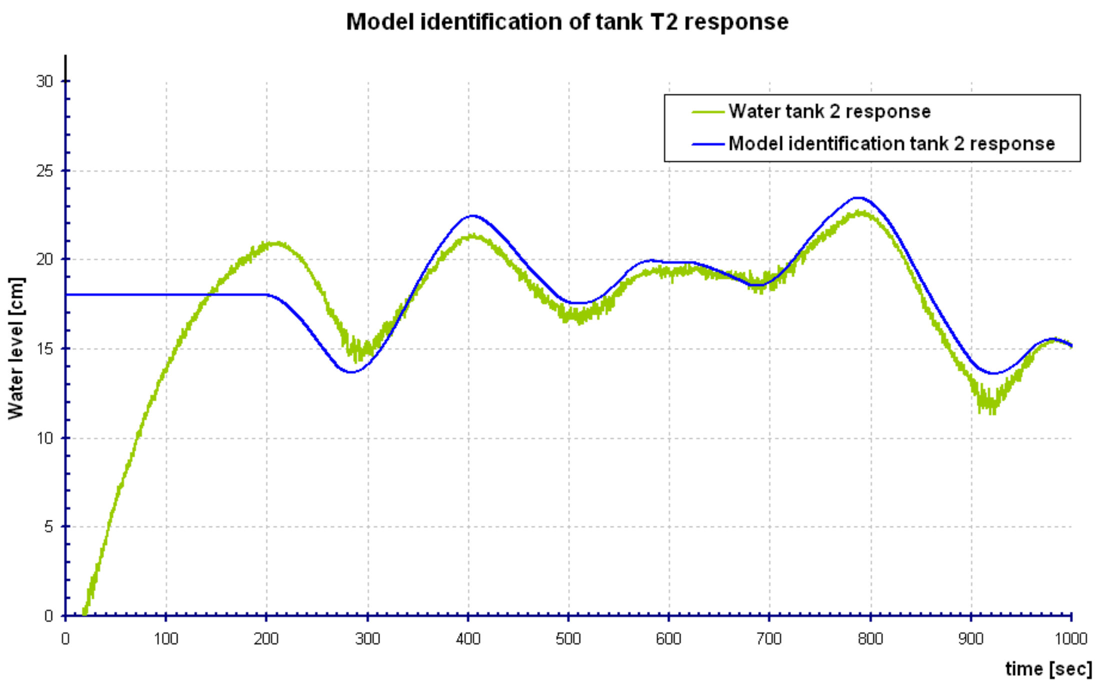


Figure 11 Tank 2 model and plant response

Coupled Tanks setup control

The Coupled Tanks control aspect covers water level control in different tanks under various conditions and setup valves configuration. For the beginning simple control schemes are considered – tank 1 water level control and tank 2 water level control. Matlab provides various analysis methods for linear systems as far as dynamics are concerned (root locus, frequency analysis tools – Bode diagrams, Nyquist plots, pole and zero maps etc.). With the information that Matlab provides about the dynamics of the system, controllers can be designed. The following sections explain how the PID controller works, and how it can be tuned.

Plant control

There are numerous control algorithms however the PID control is the most popular because of its simplicity. A general schematic of a simple control closed loop system is presented in Figure 12.

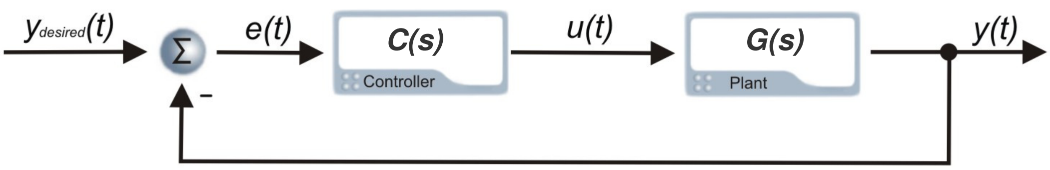


Figure 12 Simple control system – closed loop

Assuming that the plant is represented by its linear model its transfer function can be described as:

$$G(s) = \frac{B(s)}{A(s)} \tag{15}$$

where *s* is the Laplace operator. The idea of control algorithms is to find such a controller (transfer function, discrete transfer function, any nonlinear), which will fulfil our requirements (certain dynamic response, certain frequency damping, good response to the dynamic changes of the desired value etc.). Every controller's input is the *e(t)* error signal. Sometimes disturbance signals are also measured. Depending on the present and past values of the error signal, the controller performs such an action (changes the *u(t)* control signal) that the *y(t)* is as close to the $y_{desired}(t)$ value as possible at all times.

here are a lot of controller design and tuning methods. All of them consider the behaviour of the '*closed loop system*' (plant with a controller Figure 12) and provide controller parameters according to the assumed system characteristics. With the known plant transfer function *G(s)* it is possible to find satisfactory parameters of the *C(s)* controller such that the closed loop system will have the desired characteristics described by the transfer function $T_c(s)$:

$$T_c(s) = \frac{C(s)\cdot G(s)}{1 + C(s)\cdot G(s)} \quad \textbf{(16)}$$

PID controller

A PID controller consists of 3 blocks: **P**roportional, **I**ntegral and **D**erivative. The equation governing the PID controller is as follows:

$$u(t) = P\cdot e(t) + I\cdot \int e(t)dt + D\cdot \frac{de(t)}{dt} \quad \textbf{(17)}$$

$$e(t) = y_{desired}(t) - y(t) \quad \textbf{(18)}$$

With the means of the Laplace transform such a structure can be represented as a transfer function:

$$U(s) = (P + \frac{I}{s} + D\cdot s)\cdot E(s) \quad \textbf{(19)}$$

$$C(s) = \frac{U(s)}{E(s)} = (P + \frac{I}{s} + D\cdot s) = \frac{Ds^2 + Ps + I}{s} \quad \textbf{(20)}$$

Each of the PID controller blocks (*P, I* and *D*) plays an important role. However for some applications, the Integral or Derivative part has to be excluded to give satisfactory results. The Proportional block is mostly responsible for the speed of the system reaction. However for oscillatory plants it might increase the oscillations if the value of *P* is set to be too large.
The Integral part is very important and assures zero error value in the steady state, which means that the output will be exactly what we want it to be. Nevertheless the Integral action of the controller causes the system to respond slower to the desired value changes and for systems were very fast reaction is very important it has to be omitted. Certain nonlinearities will also cause problems for the integration action.

The Derivative part has been introduced to make the response faster. However it is very sensitive to noise and may cause the system to react very nervously. Thus very often it is omitted in the controller design. Derivative part output filtering may reduce the nervous reaction but also slows the response of the controller down and sometimes undermines the sense of using the Derivative part at all. Proper filtering can help to reduce the high frequency noise without degrading the control system performance in the lower frequency band.

There are several PID tuning techniques. Most often Ziegler-Nichols rules are used or a relay experiment is undertaken. Very often the closed loop system roots are analysed and set in the desired position by proper choice of *P*, *I* and *D* values. Matlab delivers a root locus tool, which helps in such designs.

Tank 1 or tank 2 water level control

As a first control task, PID water level control in tank 1 is chosen. The following exercises will guide you through the controller design, testing on the model and on the real time application.

Exercise 5 – PID control of water level in tank 1

Introduction

To design a PID controller a model of the plant is needed. For this purpose we can use a discrete model and design a discrete PID controller, or use a continuous model (equation 9) and design a continuous PID controller, however when a controller is implemented on some kind of a control unit it has to be used in a discrete form. The continuous model can also be obtained from the identified discrete model using Matlab 'd2c' function.

For the controller design the continuous model obtained in the identification experiment is used.

Task

Design a PID controller using the transfer function obtained in the identification experiment or one described by equation (9). Input the transfer function in Matlab using the '*tf*' command. The discrete model obtained in *Coupled Tanks Model Identification* section can be transformed into a continuous model before the controller design.

For instructions on controller tuning refer to the '*Matlab Guide*' manual.

Example results and comments

The root locus of the identified model with the PID control with *P = 100,I = 1.0, D = 0* is presented in Figure 13.

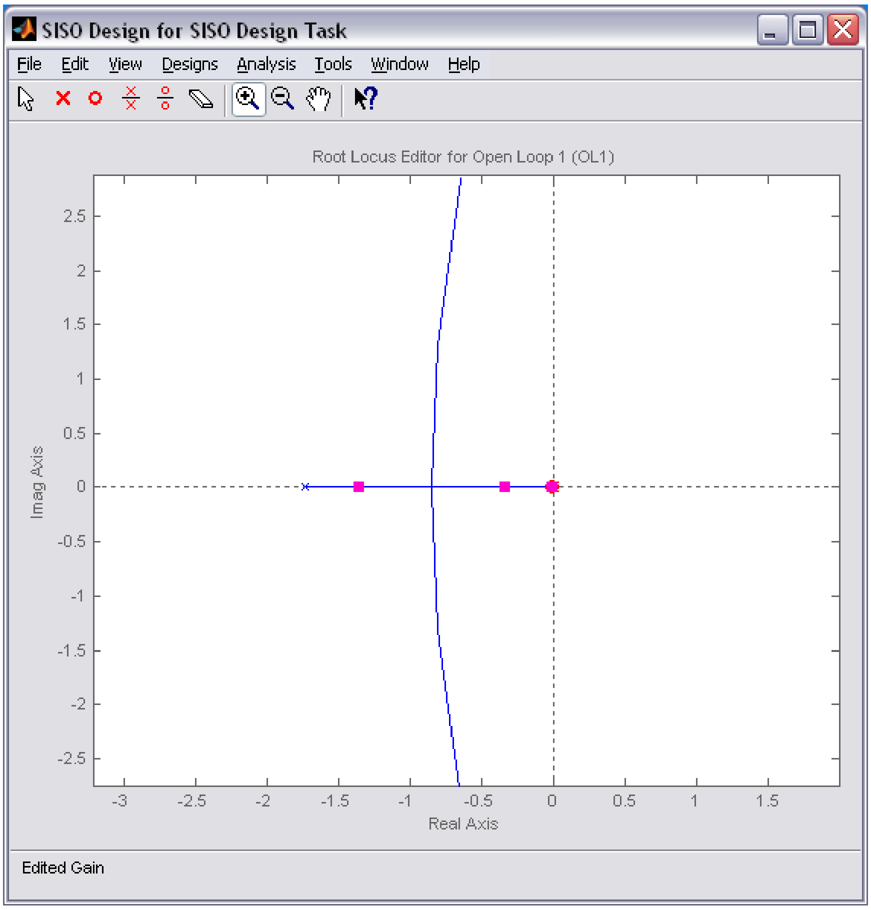


Figure 13 Root locus with PID controller

You can move the poles, zeros and the gain to obtain for example faster step response of the closed loop system. Then you can export the controller into Workspace and test it on the model of the Coupled Tanks with *CT_model_T1_PID.mdl*.

Use a sinusoidal signal for the set value of the water level $h_{desired}(t)$. Change the frequency and see how the output follows the desired value. Decrease and increase the values of the proportional, integral and derivative gains in the PID controller. See how that influences the tracking of the desired value.

COUPLED TANKS SETUP CONTROL

The controller that has been obtained in the above exercise can be tested on the Coupled Tanks setup. The results of PID control on the model are presented in Figure 14.

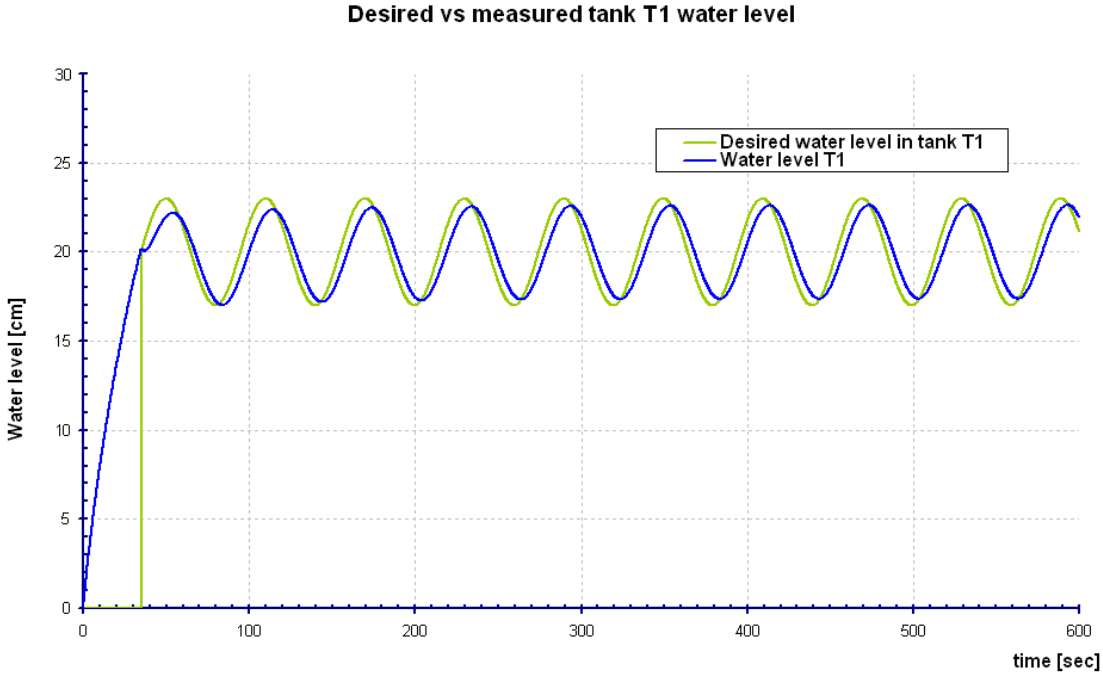


Figure 14 PID control on the model

Exercise 6 – Real-time PID control of water level in tank 1

Introduction

Open valve MVB (**leave MVE open – for safety**). The rest of the valves should be kept closed.

Just as in exercise 5 the water level in tank 1 will be controlled. This time it is tested in real-time. Anti-windup is also implemented. For this exercise use the *CT_T1_PID.mdl* presented in Figure 15.

As you open the *CT_T1_PID.mdl* you will notice that this simulation is directed to an external module, which is indicated in the upper part of the simulation window. The PID controller has been already designed, however you can change its parameters according to the results obtained in exercise 5.

Before you run the real-time simulation make sure the Coupled Tanks setup is properly connected. Refer to the '*Installation & Commissioning*' manual for more instructions on real time simulations.

Before the PID control start the tanks water level has to be brought to the working point of $u = 2.7\ [V]$, $h_1 = 16\ [cm]$, $h_2 = 20\ [cm]$.

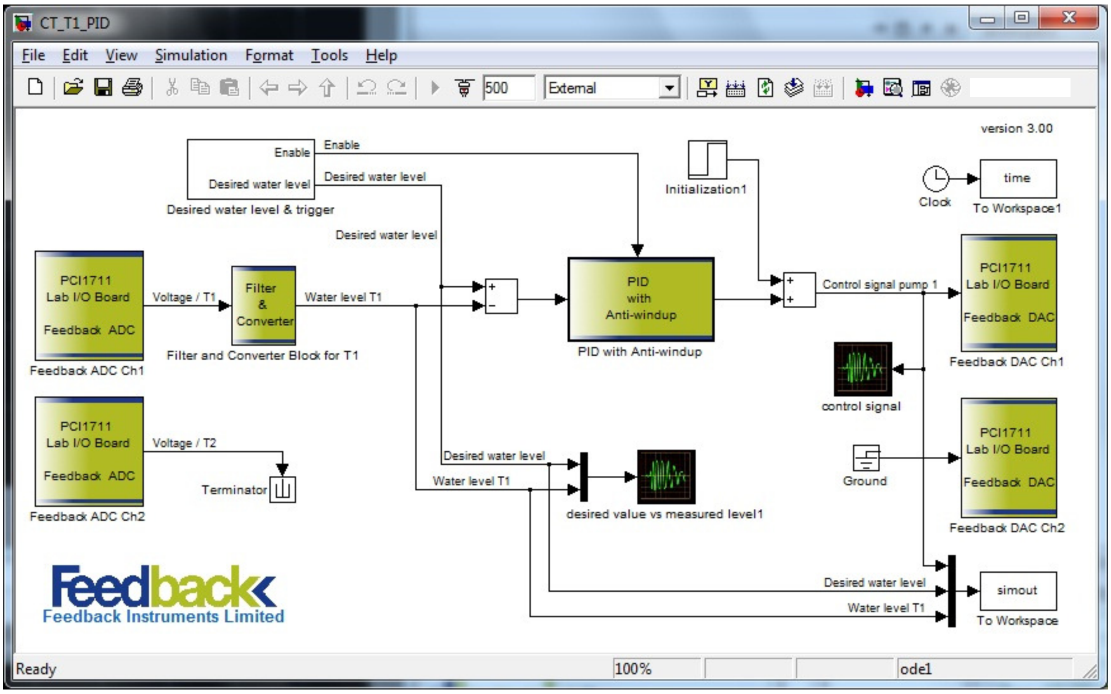


Figure 15 Real time Coupled Tanks water level control

Task

The task is to change the sinusoidal $h_{desired}(t)$ signal frequency and see how the system reacts to it. Change the controller *P,I* and *D* values to see how the signal $h_{desired}(t)$ is tracked by the system output.

You should already gain some experience in exercise 5 on how the values of these gains can be changed.

Example results and comments

Figure 16 and presents real-time simulation results of the PID water level control.

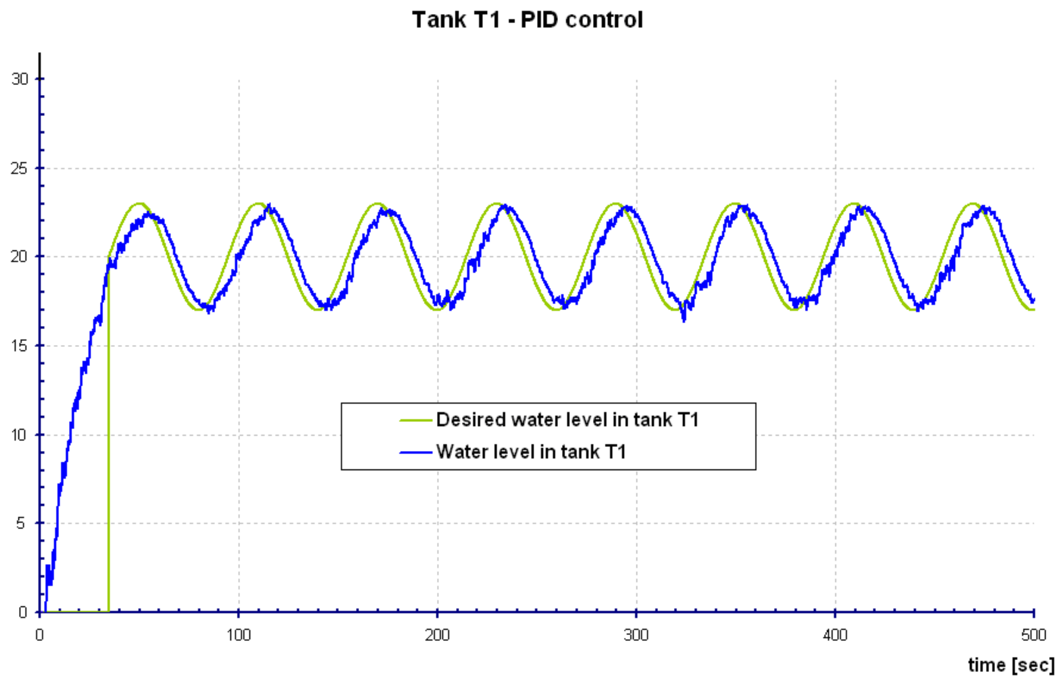


Figure 16 PID control with *P = 100, I = 1, D =0*

The result presented in Figure 16 proves how efficient controllers can be when an identification experiment is incorporated in their design.

Exercise 7 – PID control of water level in tank 2

Introduction

In previous exercises the first tank water level was controlled. This time the water level will be controlled in the bottom tank. As you may have noticed in the modelling and identification section this two models differ quite significantly. In the task of second tank water level control the water flows into the second tank through the first tank and not directly from the pump. Water level control of the two tank system is more challenging. Firstly the algorithm is tested on the model designed in Simulink.

Task

Use the *CT_model_T2_PID.mdl* to test the PID controller on the two coupled tanks. Only the second tank water level is controlled. Design your own PID controller using the model obtained in the identification exercise.

For instructions on controller tuning refer to the '*Matlab Guide*' manual.

Example results and comments

Figure 17 presents the results of an experiment where the bottom tank water level is controlled.

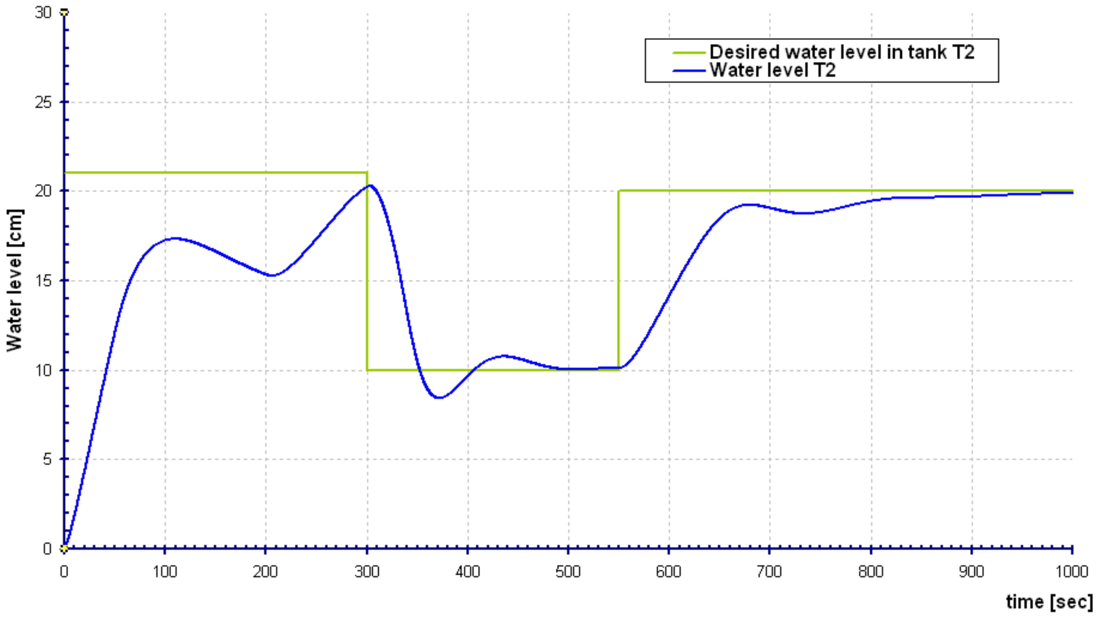
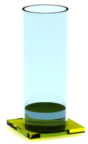


Figure 17 PID bottom tank water level control, P = 45, I = 0.38, D = 0.

Exercise 8 – Real-time PID control of water level in tank 2



Introduction

Open valve MVB (**leave MVE open – for safety**). The rest of the valves should be kept closed.

The second, bottom tank water level control is this time carried out in real-time on the plant. The controller designed in exercise 7 is used in this experiment.

Before the PID control start the tanks water level has to be brought to the working point of $u = 2.7\ [V]$, $h_1 = 16\ [cm]$, $h_2 = 20\ [cm]$.

Task

Use the *CT_T2_PID.mdl* to test the PID controller on the two coupled tanks. Again, only the second tank water level is controlled.

For instructions on controller tuning refer to the '*Matlab Guide*' manual.

Example results and comments

Figure 18 presents the results of an experiment where the bottom tank water level is controlled in real time.

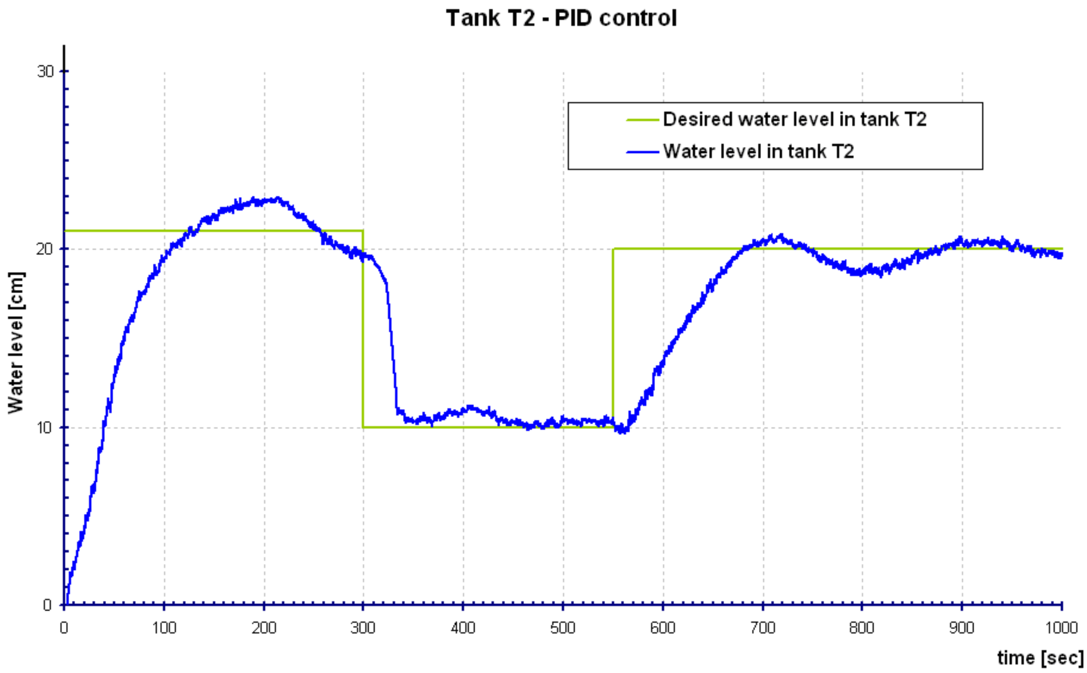
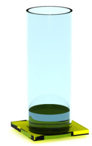


Figure 18 Real time PID bottom tank water level control

Two tanks simultaneous control

In the Coupled Tanks setup it is possible to run the same control algorithm in parallel on two top or two bottom tanks. This gives an idea of how the controllers run in parallel perform on the same setup. This also gives a chance to compare the behaviour of different controllers in real-time with the same desired value profile.

Exercise 9 – Real-time simultaneous bottom tanks water level control



Introduction

Open valves MVB, MVE. The rest of the valves should be kept closed.

In this exercise two bottom tanks water level controllers are run in parallel on the Coupled Tanks system and their performance is compared. By default their parameters are the same.

Before the PID control start the tanks water level has to be brought to the working point of $u = 2.7\ [V]$, $h_1 = 16\ [cm]$, $h_2 = 20\ [cm]$.

Task

Use the *CT_Parallel.mdl* to test the 2 PID (well tuned, badly tuned) controllers in two tank columns. The water level in tank 2 and 4 will be controlled. Observe how their performance differs when desired value profile is changed. Vary the controller parameters and use the same desired value profiles to observe how different controllers behave.

Example results and comments

Figure 19 and Figure 20 present the results of the experiment were two different controllers are run simultaneously. The water levels in tanks 2 and 4 and the control signals are compared against each other.

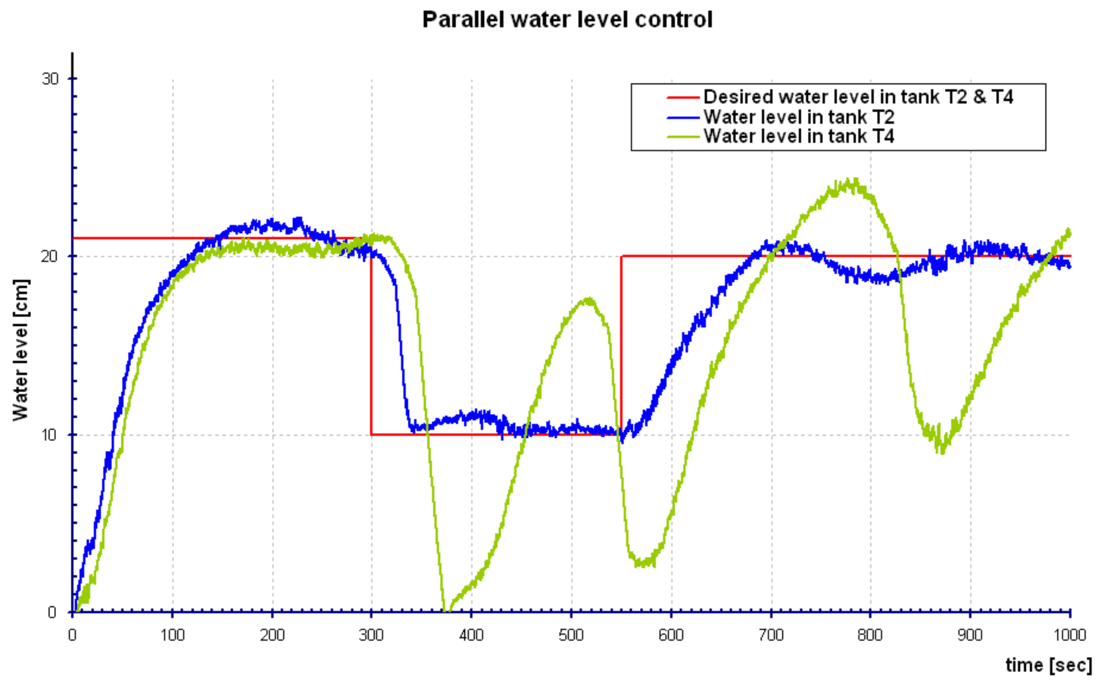


Figure 19 Simultaneous water level control, water levels

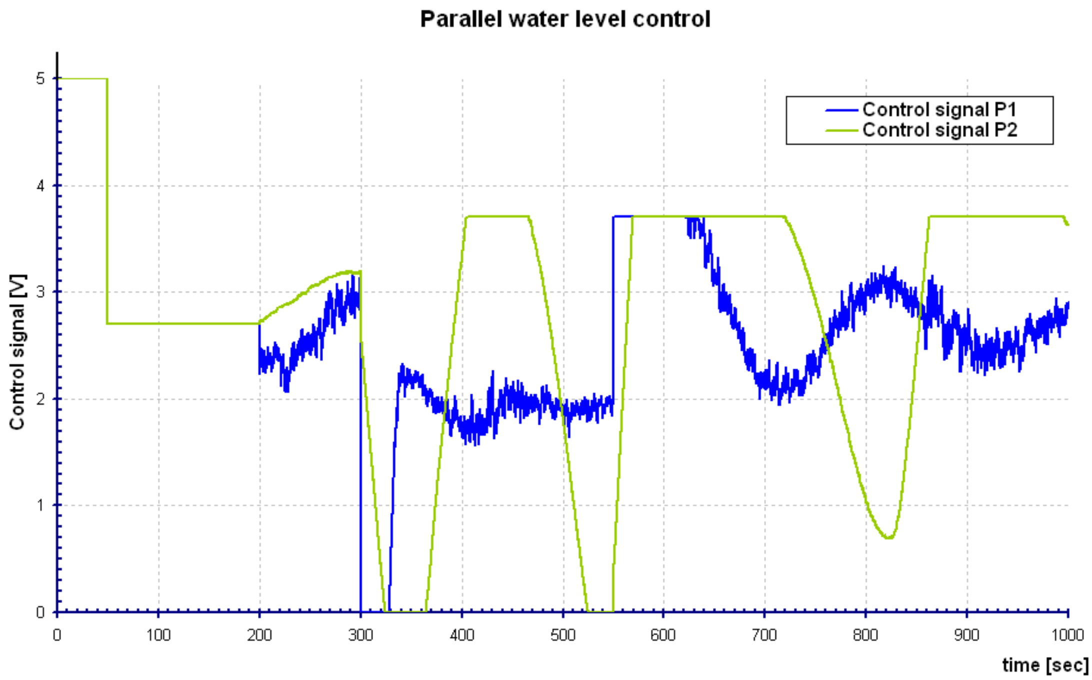


Figure 20 Simultaneous water level control, controllers outputs

Cross-coupled tanks control

Up to this section only the vertical tanks coupling has been considered, which added only a little bit more dynamics comparing to the single tank system. The output of the tank 1 was the input of tank 2 and the output of tank 3 was the input of tank 4. Now the cross-coupling is introduced. Various setup configurations are possible. The exercises and models variations considering some of the possibilities are described in the following sections.

Four coupled tanks model

Previously the two tanks dynamics were governed by (1) and (2). Now an identical parallel two tanks system is added but with internal cross coupling. The second two-tanks set dynamics are described with:

$$\frac{dh_3(t)}{dt} = -\frac{a_3}{A}\sqrt{2gh_3(t)} + \eta \cdot u_2(t), \quad (21)$$

$$\frac{dh_4(t)}{dt} = \frac{a_3}{A}\sqrt{2gh_4(t)} - \frac{a_4}{A}\sqrt{2gh_4(t)}. \quad (22)$$

The equations describing the dynamics of the coupled tanks system have to be modified when the cross-coupling is introduced:

$$\frac{dh_1(t)}{dt} = -\frac{a_1}{A}\sqrt{2gh_1(t)} + \eta \cdot u_1(t) - \frac{a_{13}}{A}\sqrt{2g\left(h_1(t) - h_3(t)\right)}, \quad (23)$$

$$\frac{dh_2(t)}{dt} = \frac{a_1}{A}\sqrt{2gh_1(t)} - \frac{a_2}{A}\sqrt{2gh_2(t)}, \quad (24)$$

$$\frac{dh_3(t)}{dt} = -\frac{a_3}{A}\sqrt{2gh_3(t)} + \eta \cdot u_2(t) + \frac{a_{13}}{A}\sqrt{2g\left(h_1(t) - h_3(t)\right)}, \quad (25)$$

$$\frac{dh_4(t)}{dt} = \frac{a_3}{A}\sqrt{2gh_3(t)} - \frac{a_4}{A}\sqrt{2gh_4(t)}. \quad (26)$$

Equations (23) to (26) describe the behaviour of the system together with the cross-couplings. The additional parameters that have been introduced describe the specific orifice size:

- a_{13} – cross-sectional area of the pipe from tank 1 to tank 3.

Figure 21 presents the diagram of the coupled system.

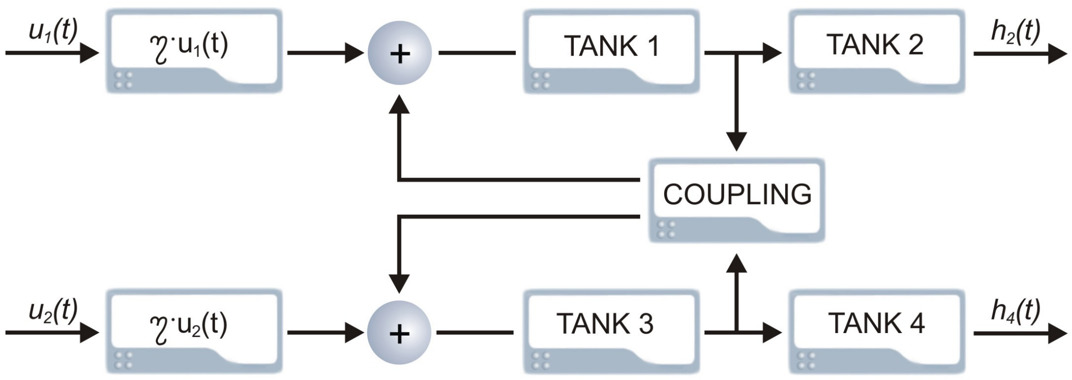


Figure 21 Tanks with dynamics coupling

The dynamics cross-coupling in the control paths may even destabilise some systems even though it was stable when no coupling was introduced. The next exercise presents how the system performance changes when the coupling is introduced in open loop.

Exercise 10 – Cross-coupling influence on the system

Introduction

In this exercise the effect of cross-coupling introduction is investigated. The cross-coupling is introduced and step changes of control signals are inflicted during water level control.

Task

Use the *CT_cross_intro.mdl* to test the influence of cross-coupling introduction. Use the manual switch to introduce the coupling. Observe the effects.

Example results and comments

Figure 22 presents the results of the experiment.

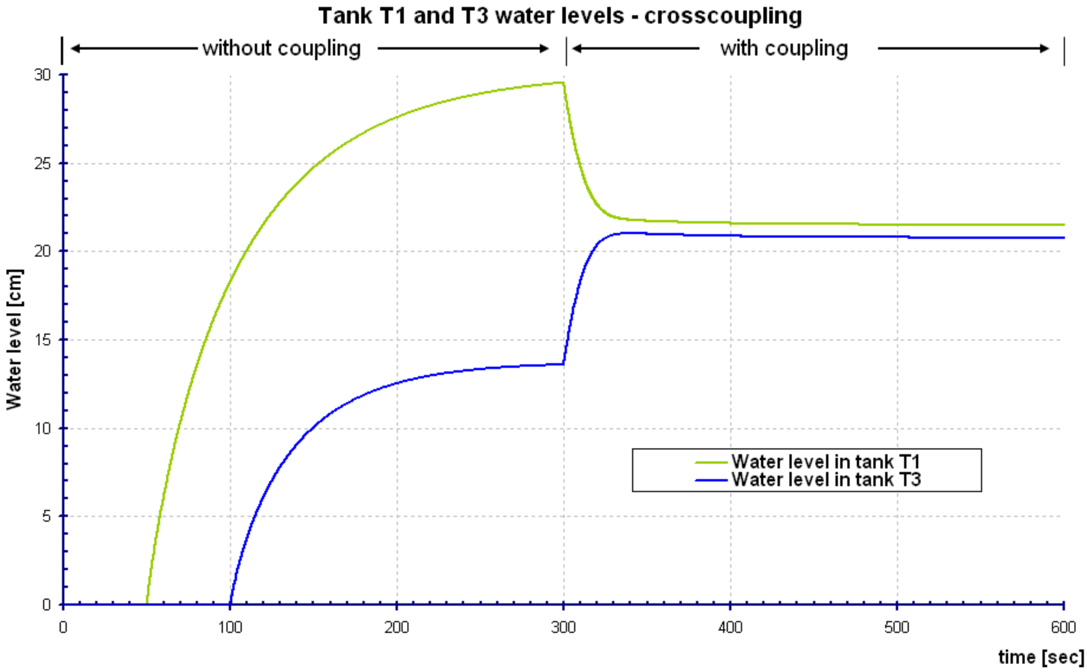


Figure 22 Cross-coupling introduction effects

Dynamics decoupling

With a significant influence of the additional dynamic paths the previously designed controllers will have to face additional difficulties during water level control. This problem is solved by introducing dynamics decoupling through the decouplers Figure 23.

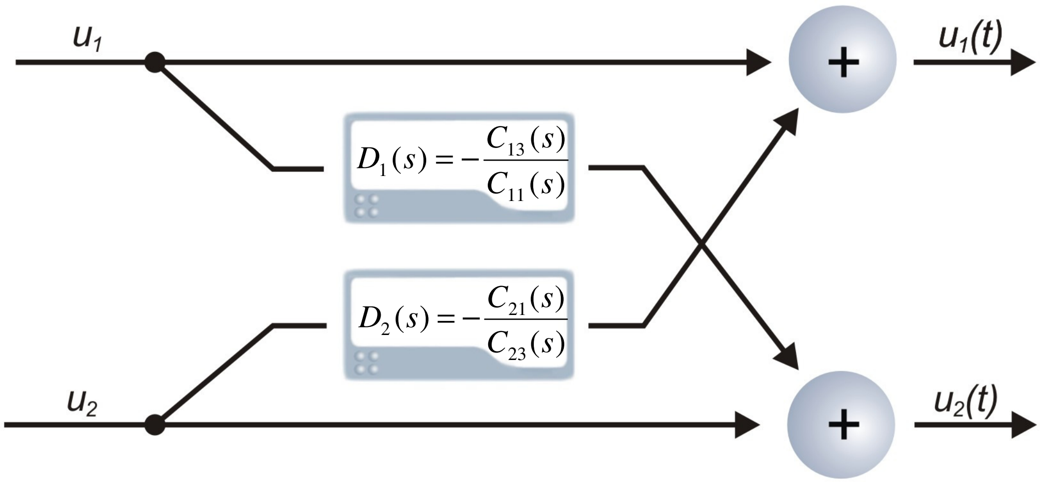


Figure 23 Decouplers introduction

The cross-dynamical paths have to be modelled. There are two ways in, which that can be performed. Either a phenomenological model is used or an identification experiment is carried out. Below the decouplers are calculated for the linearised Coupled Tanks model described by (23)-(26).

The linearised and transformed with La'place transformation system is described as:

$$\Delta H_1(s) = \frac{\eta}{s+(k_1+k_{13})} \cdot \Delta U_1(s) + \frac{k_{13}}{s+(k_1+k_{13})} \cdot \Delta H_3(s), \tag{27}$$

$$\Delta H_2(s) = \frac{k_1}{s+k_2} \Delta H_1(s), \tag{28}$$

$$\Delta H_3(s) = \frac{\eta}{s+(k_3+k_{13})} \cdot \Delta U_2(s) + \frac{k_{13}}{s+(k_3+k_{13})} \cdot \Delta H_1(s), \tag{29}$$

$$\Delta H_4(s) = \frac{k_3}{s+k_4} \Delta H_3(s), \tag{30}$$

where:

$$k_1 = \frac{a_1 g}{A\sqrt{2gh_{10}}}, \quad k_2 = \frac{a_2 g}{A\sqrt{2gh_{20}}}, \quad k_3 = \frac{a_3 g}{A\sqrt{2gh_{30}}}, \quad k_4 = \frac{a_4 g}{A\sqrt{2gh_{40}}}, \quad k_{13} = \frac{a_{13} g}{A\sqrt{2g(h_{10} - h_{30})}} \tag{31}$$

After mathematical transformations the following description is acquired:

$$\Delta H_1(s) = \frac{\eta \cdot [s + (k_3 + k_{13})]}{[s + (k_1 + k_{13})] \cdot [s + (k_3 + k_{13})] - k_{13}^2} \cdot \Delta U_1(s) + \frac{\eta \cdot k_{13}}{[s + (k_1 + k_{13})] \cdot [s + (k_3 + k_{13})] - k_{13}^2} \cdot \Delta U_2(s), \tag{32}$$

$$\Delta H_2(s) = \frac{k_1}{s + k_2} \Delta H_1(s), \tag{33}$$

$$\Delta H_3(s) = \frac{\eta \cdot [s + (k_1 + k_{13})]}{[s + (k_1 + k_{13})] \cdot [s + (k_3 + k_{13})] - k_{13}^2} \cdot \Delta U_2(s) + \frac{\eta \cdot k_{13}}{[s + (k_1 + k_{13})] \cdot [s + (k_3 + k_{13})] - k_{13}^2} \cdot \Delta U_1(s), \tag{34}$$

$$\Delta H_2(s) = \frac{k_1}{s + k_2} \Delta H_1(s), \tag{35}$$

In Exercise 11 the calculated decouplers are tested on the model in Simulink.

Exercise 11 – Dynamics decoupling on the model

Introduction

The coupling between the tanks has been introduced in the model (*CT_model_dec.mdl*). The decouplers have already been prepared. This simulation model allows for decoupling testing. When decouplers are introduced, the gain in the main paths is reduced. This has to be compensated by introducing additional gain in the main path. The necessary gain increase can be calculated knowing the evaluated gain of the decouplers, which is $k_{dg} = 0.65$. Consequently a correction gain of $k_{corr} = 1.65$ may be introduced in the main path.

Task

Use the *CT_model_dec.mdl* to test the effects of decoupling introduction on the tanks, when step changes of control signal are introduced for both pumps.

Example results and comments

Figure 24 presents results of the experiment where dynamics decoupling has been introduced on the model in Simulink.

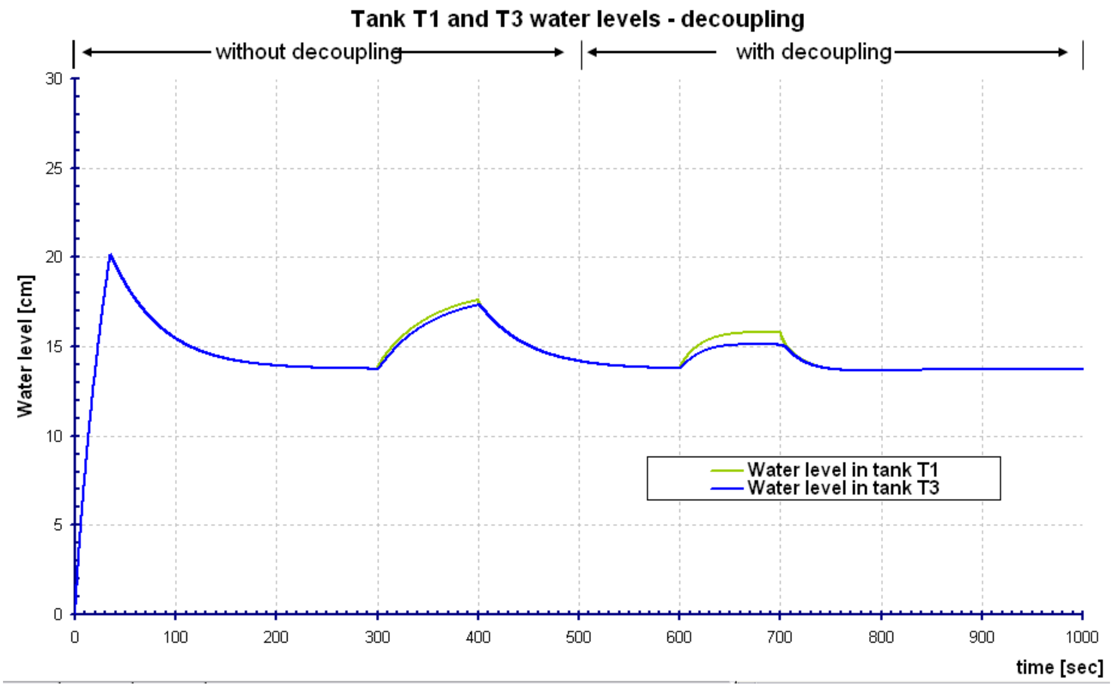


Figure 24 Dynamics decoupling on the model, T1 and T3 water levels

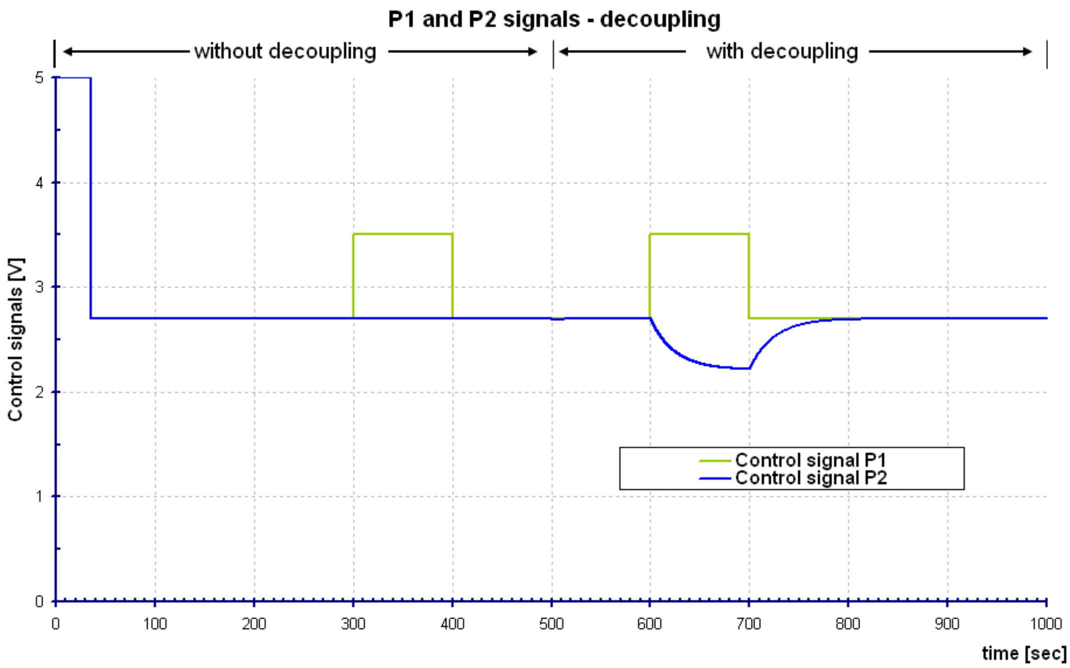


Figure 25 Dynamics decoupling on the model, control signals

Usually on the plant the coupling have to be identified. Based on the obtained models the decouplers are calculated. Exercise 12 provides an identification experiment for the cross-coupling paths. The identification has to be performed around the working points.

Exercise 12 – Cross-coupling paths identification

Introduction

Open valves MVB, MVE, MVG. The rest of the valves should be kept closed.

The identification of the cross-coupling paths is carried out using a Simulink file *CT_coupling_id.mdl*. The simulation allows for identification of the path from pump 1 to tank number 3 and from pump 1 to tank 1. The second coupling path from pump 2 to tank number 1 is assumed to be the same as from pump 1 to tank 3. The experiment lasts 1000 seconds and three signals (pump control voltage and water levels in tanks 1 and 3) are collected in the form of vectors and are available in Workspace.

Task

Run the model in Simulink. Make sure the pumps are properly submersed in the water. The water levels and the control signal are recorded.

For instructions on identification experiments refer to the '*Matlab Guide*' manual.

Identify two models (cross path and normal path) using the methods described in the mentioned manual. The suggested orders for the transfer functions are:

- oe121 for the model describing the coupling from pump 1 to tank 3,
- oe121 for the model describing the path from pump 1 to tank 1.

The obtained discrete models can be transformed into continuous equivalents according to the given instructions. The obtained models will be used in dynamics decoupling exercise 13.

Example results and comments

One might expect that the transfer functions representing the couplings should be identical. Figure 26 presents the response of the coupling path.

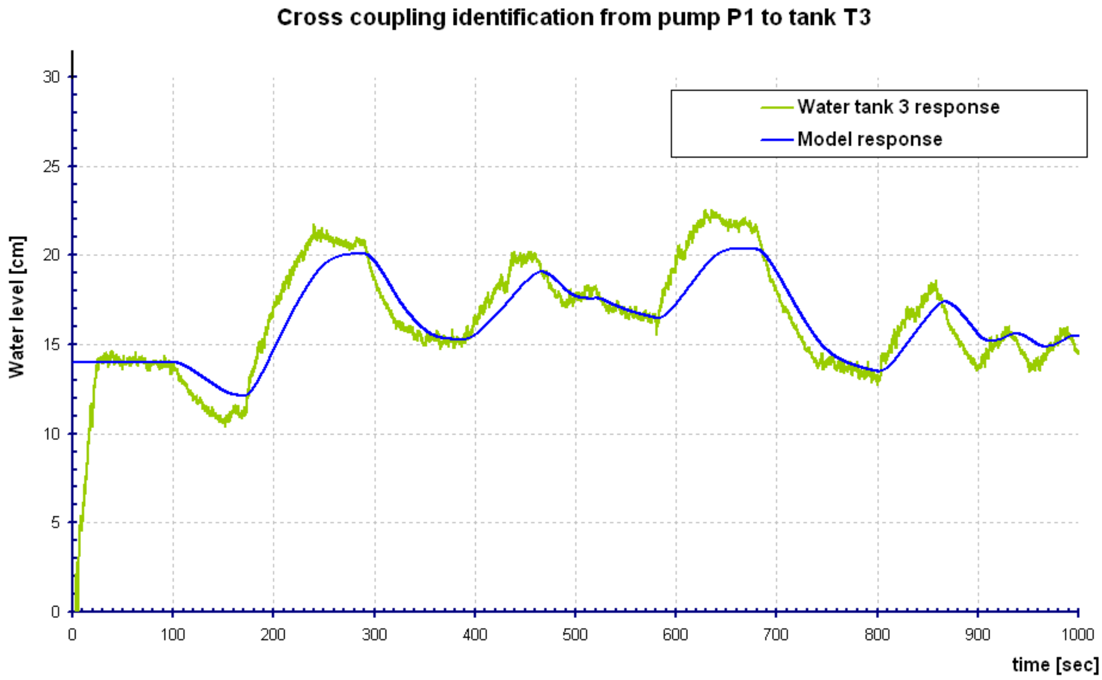


Figure 26 T3 water level response to P1 signal variation.

With the coupling identified it is possible to calculate the decouplers transfer functions according to the following equations:

$$D_1(s) = -\frac{C_{13}(s)}{C_{11}(s)}, \quad (36)$$

$$D_2(s) = -\frac{C_{21}(s)}{C_{23}(s)}. \quad (37)$$

The transfer functions C_{11} represents the model of the tank number 1. The input is the pump control voltage, u_1, and the output is the water level in the tank 1, h_1. Such model has been identified in the Model Identification section. For C_{23} the second pump control signal is the input, u_2, and tank 3 water level is the output, h_3. The transfer functions C_{13} and C_{21} represent the couplings. The first represents the coupling from pump 1 to tank number 3. The second represents the coupling from pump 2 to tank number 1 (see Figure 21). They are assumed to be the same.

When decouplers are introduced, the gain in the main paths is reduced. This may be compensated by introducing additional gain in the main path. The optional gain increase can be calculated knowing the evaluated gain of the decouplers, which is

$k_{dg} = 0.65$. Consequently a correction gain of $k_{corr} = 1.65$ is introduced in the main path.

Exercise 13 – Dynamics decoupling in real-time

Introduction

Open valves MVB, MVE, MVG. The rest of the valves should be kept closed.

In exercise 11 the decoupling was tested on the model in Simulink without control. In this exercise the decouplers are tested in real-time on the Coupled Tanks equipment with PID controllers engaged. The decouplers were calculated according to (36) and (37). The transfer functions needed for decouplers evaluation, were obtained from an identification experiment.

Before the PID controllers are started the tanks water level has to be brought to the working point of $u = 2.7$ *[V]*, $h_1 = 16$ *[cm].*

Task

Use the *CT_decoupling.mdl* to test the effects of decoupling introduction on the tanks, when step or sinusoidal changes of control signal are introduced for both pumps.

Example results and comments

Figure 27 presents results of the experiment where dynamics decoupling has been introduced on the Coupled Tanks setup.

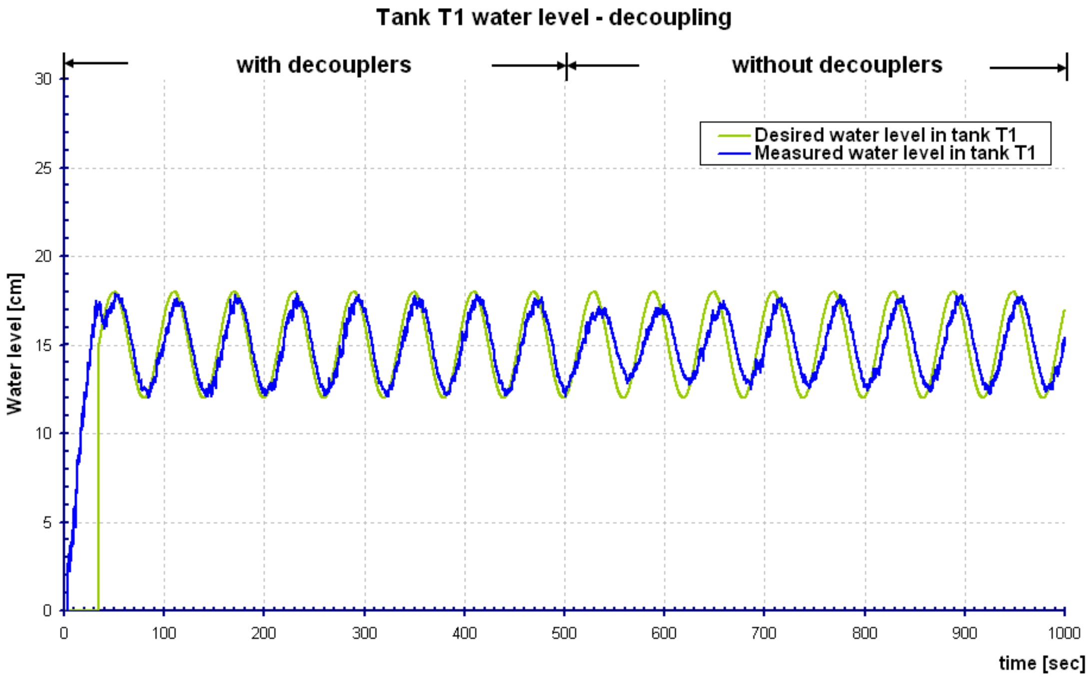


Figure 27 Dynamics decoupling in real time

Disturbance compensation

Every plant is influenced by disturbances. Some of them can be measured. Some disturbance character can be estimated. As a consequence, their influence can be compensated (feed-forward). The disturbance can be introduced in the Coupled Tanks system by opening the output valves at the bottom of the tanks. There is also a possibility of using the second pump and introduce the disturbance into tank 1 while controlling the water level in tank 1 or 2. To change the dynamics of the plant, the first pump outflow is split. Part of it is directly fed to tank 2, and the rest flows up to tank 1. The split is achieved by opening the MV A. That influences the system dynamics. The following equations describe how the dynamics change.

$$\frac{dh_1}{dt} = \gamma\eta \cdot u_1 - \frac{a_1}{A}\sqrt{2gh_1} \quad (38)$$

$$\frac{dh_2}{dt} = (1-\gamma)\eta \cdot u_1 + \frac{a_1}{A}\sqrt{2gh_1} - \frac{a_2}{A}\sqrt{2gh_2} \quad (39)$$

The working point is calculated in the following way:

(40)

$$0 = \gamma\eta \cdot u_{10} - \frac{a_1}{A}\sqrt{2gh_{10}} \qquad\qquad h_{10} = \frac{1}{2g}\cdot\left(\frac{A\gamma\eta \cdot u_{10}}{a_1}\right)^2$$
$$0 = (1-\gamma)\eta \cdot u_{10} + \frac{a_1}{A}\sqrt{2gh_{10}} - \frac{a_2}{A}\sqrt{2gh_{20}} \quad \Rightarrow \quad h_{20} = \frac{1}{2g}\cdot\left(\frac{A\eta \cdot u_{10}}{a_2}\right)^2 \qquad \textbf{(41)}$$

Linearisation and La'place transform yields:

$$\Delta H_1(s) = \frac{\gamma\eta}{s + \left(\frac{a_1}{A}\right)^2 \cdot \frac{g}{\gamma\eta u_{10}}} \cdot \Delta U_1(s) = K_1(s)\cdot \Delta U_1(s) \qquad \textbf{(42)}$$

$$\Delta H_2(s) = \frac{(1-\gamma)\eta}{s + \left(\frac{a_2}{A}\right)^2 \cdot \frac{g}{\eta u_{10}}} \cdot \Delta U_1(s) + \frac{\left(\frac{a_1}{A}\right)^2 \cdot \frac{g}{\gamma\eta u_{10}}}{s + \left(\frac{a_2}{A}\right)^2 \cdot \frac{g}{\eta u_{10}}} \cdot \Delta H_1(s) = K_2(s)\cdot \Delta U_1(s) + K_3(s)\cdot \Delta H_1(s) \qquad \textbf{(43)}$$

The system structure with disturbance is presented in Figure 28.

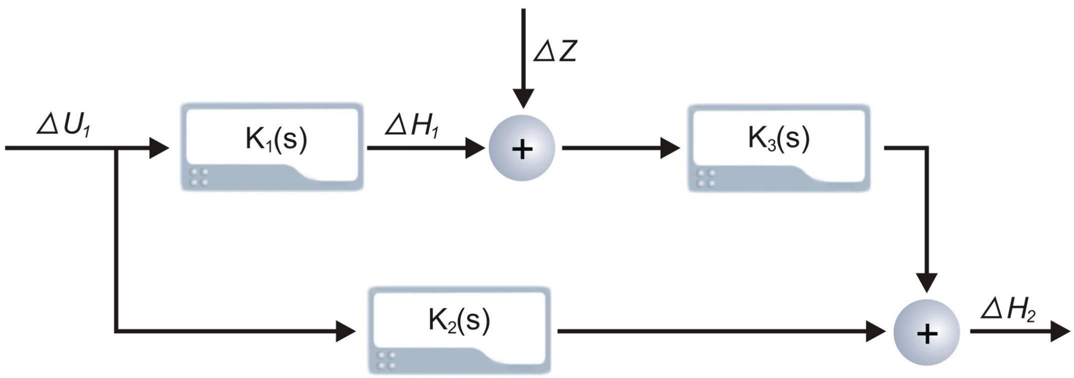


Figure 28 System structure with disturbance

The whole system with disturbance is:

$$(\Delta H_1(s) + \Delta Z(s))\cdot K_3(s) + K_2(s)\cdot \Delta U_1(s) = \Delta H_2(s) \qquad \textbf{(44)}$$
$$\Delta H_1(s) = K_1(s)\cdot \Delta U_1(s) \qquad \textbf{(45)}$$

(46)

$$(K_1(s)\cdot\Delta U_1(s)+\Delta Z(s))\cdot K_3(s)+K_2(s)\cdot\Delta U_1(s)=\Delta H_2(s)$$
$$\Downarrow$$
$$(K_1(s)K_3(s)+K_2(s))\cdot\Delta U_1(s)+K_3(s)\Delta Z(s)=\Delta H_2(s) \qquad \textbf{(47)}$$

The structure with the disturbance compensator is presented in Figure 29.

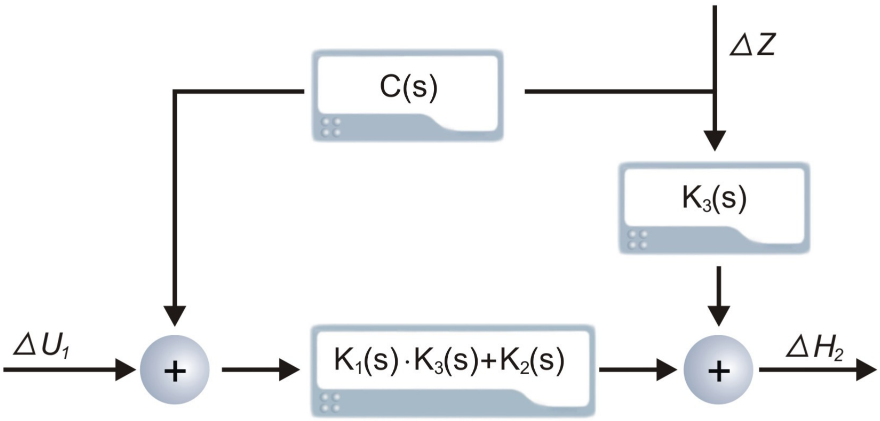


Figure 29 Disturbance compensator

Thus the compensator should be:

$$(\Delta U_1(s)+C(s)\Delta Z(s))\cdot(K_1(s)K_3(s)+K_2(s))+K_3(s)\Delta Z(s)=\Delta H_2(s) \qquad \textbf{(48)}$$

$$(K_1(s)K_3(s)+K_2(s))\cdot\Delta U_1(s)+\left(C(s)(K_1(s)K_3(s)+K_2(s))+K_3(s)\right)\Delta Z(s)=\Delta H_2(s) \qquad \textbf{(49)}$$

$$C(s)(K_1(s)K_3(s)+K_2(s))+K_3(s)=0 \qquad \textbf{(50)}$$

$$C(s)=\frac{-K_3(s)}{K_1(s)K_3(s)+K_2(s)}=\frac{-\frac{1}{\eta}}{1+s\cdot\frac{1-\gamma}{\left(\frac{a_1}{A}\right)^2\cdot\frac{g}{\gamma\eta u_{10}}}} \qquad \textbf{(51)}$$

Exercise 14 – Disturbance compensation on the model

Introduction

The disturbance compensator is introduced in model (*CT_model_comp.mdl*). This simulation model allows to test its influence.

Task

Use the *CT_model_comp.mdl* to test the effects of disturbance compensation. Use the manual switch to turn the compensator on and off.

Example results and comments

Figure 24 presents results of the experiment where disturbance compensation has been introduced on the model in Simulink.

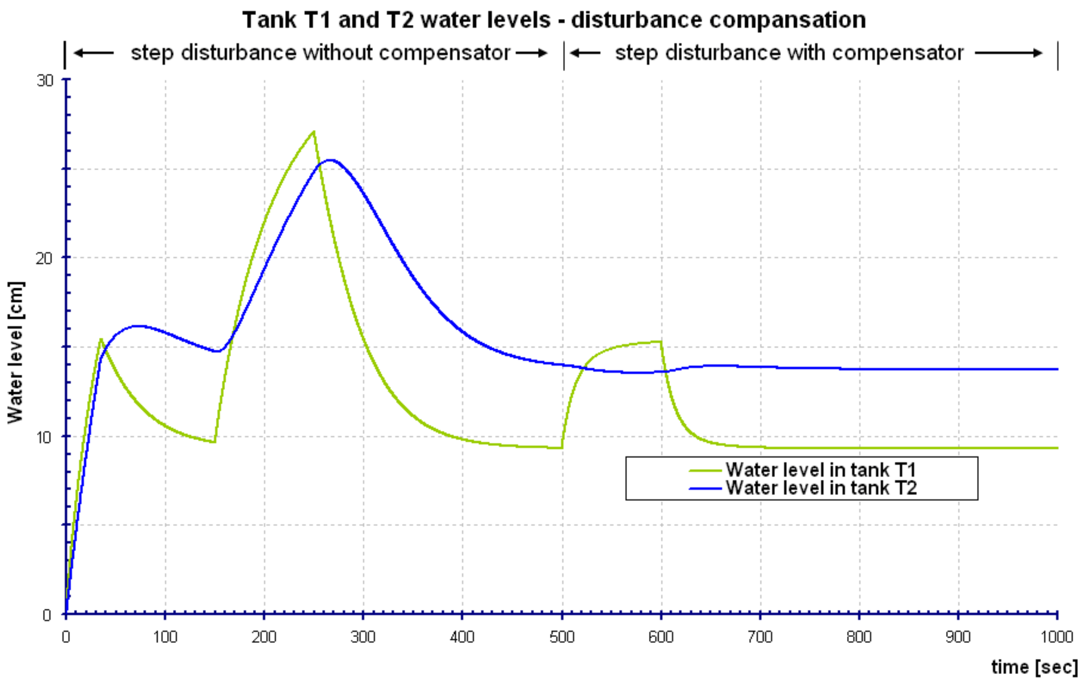


Figure 30 Disturbance compensation on the model

Exercise 15 – Real-time disturbance compensation with PID control

Introduction

Open valves MVA, MVB, MVF, MV1 and MV2. The rest of the valves should be kept closed.

The disturbance compensator is introduced in real-time (*CT_dis_comp.mdl*). This simulation model allows to test its influence.

Before the PID control starts the tanks water level has to be brought to the working point of *u = 2.7 [V]*, h_1 *= 16 [cm]*, h_2 *= 20 [cm].*

Task

Use the *CT_dis_comp.mdl* to test the effects of disturbance compensation. Use the manual switch to turn the compensator on and off.

Example results and comments

Figure 24 presents results of the experiment where disturbance compensation has been introduced on the model in Simulink.

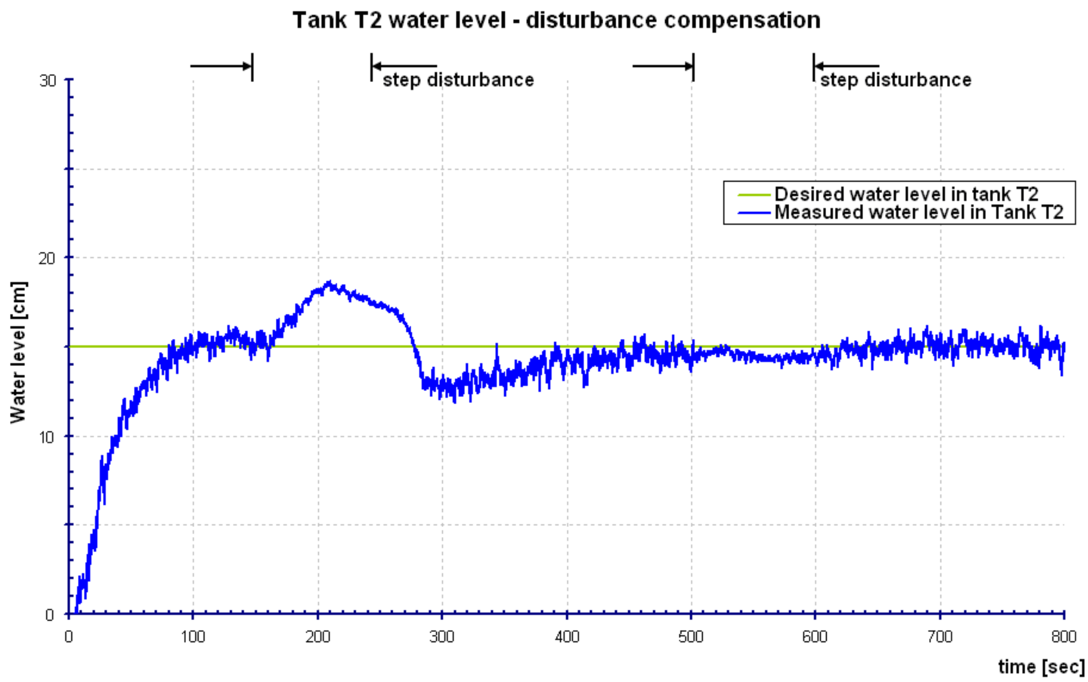


Figure 31 Disturbance compensation in real time